A REPLACEMENT MODEL FOR EQUIPMENT WITH A CONSTANT OPERATING REQUIREMENT: A SOLUTION TO A DEPARTMENT OF DEFENSE ALLOCATION PROBLEM FOR TACTICAL MILITARY VEHICLES

by

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ABSTRACT

Equipment which must maintain some minimum level of operations poses a difficult problem in determining the economic service life. This is especially true when this equipment is subject to a deterioration of effectiveness due to age. It is the purpose of this paper to investigate this problem in its specific application to tactical military vehicles. This application poses institutional and organizational constraints which must be considered.

Using an inventory approach, the model is developed for the batch procurement situation. The embedded functional relationships are identified and introduced into a cost equation which can be solved for the economic service life. This basic model is then expanded to include the situation of continuous procurement, attrition, present value comparisons, and salvage value.

In the investigation an attempt is made to define the parameters by empirical analysis. However, the data were not available to support this effort. Approximations are derived in order to exercise the model in a somewhat realistic environment. These parameters were introduced into the computer program which was designed to solve the model.

Sensitivity analysis was conducted to determine the model's sensitivity to a change in the parameters. This analysis shows that the present replacement policy is the economic replacement policy. However, small changes could have drastic results. The situation where the personnel varied with the age of the supported equipment was addressed. The analysis clearly indicates that this area should receive detailed investigation.

The study concludes that the out-of-service time of tactical military vehicles should be carefully investigated. This can only be accomplished if out-of-service is defined as the total time the equipment is not available to perform its intended mission. Also, that the personnel situation should be thoroughly investigated to determine the effects of the age of the supported equipment on the number of personnel required. When the correct parameters are available, the model should be exercised to ascertain if the current replacement policies should be changed.

A REPLACEMENT MODEL

FOR

EQUIPMENT WITH A CONSTANT OPERATING REQUIREMENT:

A SOLUTION TO A DEPARTMENT OF DEFENSE

ALLOCATION PROBLEM

FOR TACTICAL MILITARY VEHICLES

by

Lawrence Patrick Hart

PREFACE

During my recent tour of duty in the Republic of Viet Nam, I was assigned as the Commanding Officer of Wing Equipment and Repair Squadron 17. This unit was responsible for providing the fourth echelon maintenance for the non-aircraft motorized equipment of the Marine Aircraft Wing. From this experience, I became intrigued with the problem of maintaining tactical military vehicles. I was firmly convinced that the current replacement policies were in error and that there must be some better way to compute replacement cycles. My investigation has revealed that the problem is far more complex than I thought. But specialized techniques can be used to provide more accurate replacement information. I hope that my investigation will produce sufficient interest so that data from which more accurate functional relationships can be derived will be collected.

I wish to express my appreciation for the invaluable assistance my advisors, Mr. Jerry J. Shipley and Mr. John R. Transue, have given me. Without their advice and criticisms the results would have been meaningless. However, all errors contained in the paper are solely mine.

I must also acknowledge the patience, support and understanding shown by my wife and children during the development of the paper, and especially for my wife's efforts in preparing this manuscript.

Lawrence F. Hart
28 December 1968

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CHAPTER I

INTRODUCTION

> "The majority of durable goods requires during their service life a flow of maintenance expenditures, which as a rule rises irregularly with age and use. Most of them suffer a deterioration in the quality of their service as time goes on." [1]

Purpose

It will be the purpose of this paper to analyze a very simple military problem, the replacement of tactical military vehicles. Replacement decisions are normally based on an analysis which makes use of replacement theory. Replacement theory itself is a simple application of the theories of the allocation of scarce resources among competing objectives. In the civilian application, there is usually the profit standard and motive which can be used as a basis for the allocation problem. In the military, there is no profit standard that can be employed and some other standard must be used as a basis. Normally, some measure of effectiveness is used

Background

This problem is more than just a theoretical application of economic thought and tools of the trade. A solution to this problem

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[1] George Terborgh, Dynamic Equipment Policy, (McGraw-Hill, New York, 1949), p. 2.

could well remedy an existing shortcoming in our military posture. For the third time in the past 25 years military forces of our nation have been committed in support of our national objectives. In each case, the forces were deployed with the equipment on hand. This is the normal situation and we should expect that our organizational and planning procedures would provide the proper equipment and level of readiness. The equipment on hand should be reasonably up-to-date and sufficiently reliable to provide the expected service. If this is not true, then we can suspect some fault in the planning system and procedures.

Tactical military vehicles appear to be one area where reliability is in question. The number of available vehicles steadily declined when subjected to the extreme operating environments experienced in combat. This was true in World War II and Korea as well as in Viet Nam. While some of the out-of-service time is naturally explained by the combat attrition, this alone is insufficient to explain the increased out-of-service time that has been experienced. The usual maintenance structure nominally expected to maintain a satisfactory level of operations was available.[2] In the early months of the deployments,

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[2] A comprehensive maintenance and repair structure has been established to provide a nominal level of daily availability. This maintenance and repair structure consists of five separate echelons of effort, labeled First through Fifth Echelon Maintenance.

First Echelon: This maintenance is performed by the user or operator of the vehicle in providing the proper care, cleaning, lubrication, adjustment, and minor repairs.

Second Echelon: This maintenance is performed by specially trained personnel (i.e., mechanics) and is usually limited to adjusting, tuning and replacement of minor assemblies.

Third Echelon: This maintenance is performed by mechanics using shop and test equipment. They repair subassemblies and replace major assemblies.

Fourth Echelon: This maintenance is performed by mechanics using permanent or semi-fixed shops to repair subassemblies, assemblies, and

a lagging supply system could be identified as a major contributor to the lower availability. With time, the supply system responded to the increased demands with more and better support, but the availability did not return to normal. Other than the supply portion of the problem, basic reliability would appear to be the major area to be investigated.

One of the major difficulties encountered in an analysis of tactical military vehicles is the variety of different models. A critical examination of the effectiveness of a tactical military vehicle may shed some light on the problem. In this paper, a continuing example of a 5 ton truck, M 52, general purpose cargo (and allied applications), will be used. Although the example will be limited to this one type of vehicle, it is clearly evident that the other types of motorized equipment (i.e., generators, compressors, engineering equipment, airfield crash and survival equipment, etc.) would present similar cases.

Existing Replacement Methodology

In considering the allocation problem, it is necessary to combine the various substitutions which can be made between the competing factors and applications. At present, the commodity manager (the office responsible for planning and programming the purchase and replacement of an item of equipment) for motor vehicles is the U. S. Army. This office uses the following model to compute the replacement

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major items for return to lower echelons and to the supply system.

Fifth Echelon: This maintenance is performed by highly trained mechanics and uses production and assembly line methods for rebuilding major items, assemblies, accessories, and parts.

This definition is contained in TM-11240-15/1A USMC published by Headquarters U. S. Marine Corps, Jan. 1967.

requirements for tactical military vehicles. The optimum life is found by computing the value of the cost-effectiveness per unit for each interval. The associated n, for the minimum cost interval, will then give the optimum life for the equipment under consideration.

$$E_n = \frac{1}{n}\left[\frac{A}{n}\sum_{i=1}^{n}\frac{1}{q_i} + \sum_{i=1}^{n}\left[\frac{C_i}{n-(i-1)}\sum_{j=1}^{n}\frac{1}{q_j}\right]\right]$$

where E_n = Average cost-effectiveness per interval after n intervals of use. (Dollars per unit effectiveness)

A = Item acquisition cost. (Dollars per unit)

C_i = Total maintenance cost in interval i. (Dollars per unit)

q_i = Effectiveness in interval i. (Pure fraction)

This methodology for the calculation of replacement requirements is sensitive to the present age distribution of the current inventory. Wearout probabilities by item age and a yearly accidental loss rate are applied to the age distribution of the in-use inventory to determine the replacements required for the following year. Wearout is determined by physical deterioration which results in decreasing efficiency and increasing maintenance costs with time until a point is reached when replacement is a more profitable course than further repair. Wearout occurs at effective life, which is the age at which the average cost per unit of effectiveness per use period is least. The minimum cost over time is obtained by replacement at the point of effective life with an identical new item which in turn may be expected to achieve the same effective life and the same minimum cost-effectiveness.

Acquisition cost is amortized equally over the number of intervals, n, being considered. Maintenance costs are amortized in equal increments starting with the period in which they are incurred through all remaining periods. The total of the costs to be amortized in a given interval is adjusted by the effectiveness during the interval. This cost-effectiveness ratio is the true cost per unit of perfect effectiveness in each interval. The measure of effectiveness is the probability of mission success. A successful mission depends on the joint probability that a vehicle is available to start the mission and that it completes the mission without breakdown. This joint probability is the product of the reliability and the availability.

This model was developed in response to a requirement for determining accurate requirements and as a means of properly justifying requests for these replacements before the Congress. The methodology is a system for predicting requirements based on anticipated losses to an inventory. Losses are postulated after considering the age or mileage distribution of the current in-use inventory and the anticipated usage of this inventory in future years. An accidental loss rate was developed as a function of the age of the vehicle or item of equipment. Wearout was determined to be a function of the age and yearly usage.[3] Physical deterioration results in decreasing efficiency and increasing maintenance costs over time until item replacement is more economical than further repair. Effective life is defined as the age when the average cost per unit of effectiveness per period is

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[3]Virginia W. Perry and others, "A Replacement Requirement Methodology for Procurement of Army Equipment," Vol. I & II, U.S. Army Logistics Management Center, Fort Lee, Va., July 1967

minimized. By replacing equipment when the average cost per use period is a minimum, major item procurement and maintenance costs are minimized.

This model was developed as a peace time methodology, to be supplemented by a war-time modification. The methodology considers the aging-replacement cycle as a Markov process where no item ever escapes from the inventory, but is always returned upon the occurrence of loss to the first age or mileage category. The Markov process requires a transition matrix based on the accidental loss rate, wearout rate, and yearly usages rates. This methodology provides an extremely accurate prediction for replacement requirements. Although the model has inputs of procurement costs, maintenance costs (including parts and labor during each age or usage period) and a measure of effectiveness, it is not a fully accurate tool for determining the economic replacement life for these items.

Analytical Shortcomings

The shortcoming of the model stems from its failure to provide for any reduction in the total number of vehicles required when there is increased availability produced as a result of more frequent replacement. It assumes that the in-use inventory will remain constant and seeks the minimum cost of maintaining this status. A more complete analysis would be to consider the entire system. In other words, the model should go beyond the in-use inventory basis and provide a similar cost-effectiveness comparison in terms of the required capability.

For example, if a unit presently has a requirement (i.e., Table of organization and Equipment, TO&E, allowance) for 100 vehicles, they could normally be expected to provide some level, say 80 percent of daily output or availability. On this basis, a daily operating requirement of 80 vehicle units of capability could be assigned. If some means could be found to increase the availability to 90 percent, this same 80 vehicle capability can be satisfied with an assignment of only 89 vehicles (TO&E allowance). Therefore, if the selection of the economic service life can cause an associated decrease in the total in-use inventory required, it must be explicitly included in the model. Explicit inclusion is doubly important when the cost of the associated personnel is computed.

Goal of This Analysis

Before commencing an analysis, it is necessary to define the problem in more precise terms. From the example of the number of trucks required to produce 80 vehicle units of capability each day, it is clear that the problem encompasses more than the mere computation of the number of replacements that will be needed. The analysis cannot make the normal assumption that the output, and therefore the number of machines is constant. For if the analysis assumes a given requirement for a number of vehicles, there will be no opportunity to take advantage of some trade-off between the total inventory of vehicles and a possible increase in the availability due to variations in the average life of the equipment on hand. With a fixed number of trucks, one could only hope to achieve the highest possible availability

within the constraints of men, money and time. One of the constraints would have to be some minimum acceptable level of availability. Otherwise, it is possible that the units using this equipment would not be able to fulfill their assigned missions. If both a cost constraint and a minimum level of availability are specified, it is possible that there may be no equilibrium solution.

Normally the military requirement is thought of as a fixed number of vehicles established by the TO&E's. However, if the situation is viewed as one where there is a certain output required each day, measured in vehicle units, then the true military requirement can be stated in terms of capability. With the military requirement, stated in units of capability, fixed in terms of what must be accomplished rather than in the present terms of a fixed number of vehicles, it will be possible to introduce sufficient flexibility to allow a more complete analysis. This approach has an additional advantage in that it will encompass the whole system. Dale W. Jorgenson states that one of the most common shortcomings of the current replacement models is ". . . failure to compute the cost of the total inventory required to support the system requirement."[4] Since the mission of the military is to equip and maintain forces capable of being deployed for sustained combat, the use as the requirement of some measure of capability to perform what must be accomplished appears to be reasonable. If a constraint is introduced which provides that the capability must equal or be greater than the task expected and a minimum cost solution is sought,

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[4] Dale W. Jorgenson and others, Optimal Replacement Policy, Rand Corp., (Amsterdam: North Holland Press, 1967), p. 6.

the result should be the cost-effective solution.

The problem, then, is the development of a comprehensive model which will encompass the entire system using capability to perform the mission as a standard rather than the usual fixed number of vehicles. The model must be capable of optimizing the system in its entirety and its cost in terms of the replacement cycle for the equipment.

Summary

In summary, the current replacement methodology for tactical military vehicles does not appear to be satisfactory because of its failure to encompass the whole system. The current practice of stating the military requirements in terms of a fixed number of vehicles rather than the capability to perform the mission precludes a complete economic analysis of the system. The substitution of capability for the fixed number of vehicles will introduce sufficient flexibility to allow meaningful economic trade-offs between the competing factors. A model will be developed which will encompass the entire system and seek a least cost solution. The time period of this equilibrium point will be the economic replacement life for these vehicles.

CHAPTER II

DEVELOPMENT OF THE MODEL

> "When a complex equipment fails because of a failure of one of its components, it is not customary to throw out the whole equipment, but to replace the failed component with a new one. Thus, in place of equipment with all components aged x, we have equipments with n-1 parts aged x and one part aged P. Then the equipment is used again until a part fails, either one of the original components or the new one. This goes on again and again, each time a failed component being replaced by a new one. This has the effect of mixing the ages of the components until they can be mixed no further, and as rigorously provided in renewal theory, an essentially steady state is reached in which the age distribution remains relatively constant and replacement of parts goes on at a relatively uniform rate."[5]

Purpose

The development of a replacement model for equipment which has a constant operating requirement requires that the various controlling relationships be identified. Of principal interest to this economic analysis is the cost and the effectiveness. In order to determine the cost, we must know the attrition, the personnel requirements, supporting costs, as well as the cost of the items which will be used as replacements. However, before these basic considerations are investigated, it may be well to review some fundamental ideas.

Most authors, when considering replacement theory, make the assumption that the output per machine is constant.[6] However, in the case

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[5] Joseph F. McCloskey and John M. Coppinger, Operations Research for Managers, (Baltimore: John Hopkins Press, 1956), p. 336.

[6] Vernon L. Smith, "Economic Equipment Policies: An Evaluation", Management Science, (Vol. 4, 1958), p. 21.

where the equipment suffers a deterioration of effectiveness with age or use, this assumption can lead to erroneous policy conclusions. A review of the proposed replacement model in light of basic economic theory will highlight the differences.

Fundamental to this analysis is the fact pointed out by Smith, "Milling machines are not replaced by milling machines, milling capacity is replaced by milling capacity."[7] For a machine that produces 100 percent effectiveness until it fails and is replaced, it is possible to consider a machine for machine replacement. Otherwise, it is necessary to take the decreasing effectiveness into consideration. When decreasing effectiveness is considered, production theory is usually used to determine the trade-offs between the factors of production. In the case of tactical military vehicles, the factors would seem to be vehicles, labor, repair parts, and working space. But even with the optimum mix of these factors, the age of the equipment will determine the availability of the vehicles. An optimum mix of these factors is achieved when the ratios of their marginal product to their factor prices are equal.[8]

With military vehicles, the production function is by a monopolist who consumes his own product while producing another product, defense, with no quantifiable price. The lack of identifiable prices complicates the solution of the problem, but does not relieve us of "cost consciousness" or "profit maximizing" behavior. "Project Prime" states

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[7] Vernon L. Smith, Investment and Production, (Cambridge: Harvard University Press, 1961), p. 133.

[8] R. D. G. Allen, Mathematical Analysis for Economists, (New York: St. Martin's Press 1964), p. 372.

that "cost effectiveness is no more than an application of simple logic. What it seeks to determine is very straight forward -- either the greatest product obtainable from a given level of resources or the least amount necessary to achieve a particular goal."[9]

The consideration of effectiveness for vehicles can allow an indirect substitution of vehicles (capital) for labor and repair parts. W. E. G. Salter states that, "Changes in the relative prices of labor and capital are strongly influenced by the cheapening of capital goods relative to wages resulting from technical progress in the capital-goods industries. This induces substitution of capital equipment for labor throughout the economy even when the rate of interest is constant."[10] Although technical progress in the tactical military vehicle area is a minor consideration, increasing military wages produce a change in the relative factor prices.

Full consideration of any such substitution, and the potential savings that may be effected, is sometimes retarded by institutional and organizational constraints of the Department of Defense. Where feasible, an attempt is made to circumvent these constraints in the development of the model. However, the problems of putting into effect changes in personnel levels, supplies, and new equipment are complicated by the budget process wherein these items are contained in different appropriations sub-heads. A decrease in one sub-head with an

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[9] "A Primer on Project Prime", Department of Defense (OASD) Comptroller, Nov. 1966.

[10] W. E. G. Salter, Productivity and Technical Change, (London: Cambridge University Press, 1960), p. 45.

associated increase in another would seem to be an easy matter to resolve. But the former may be approved while the latter is disapproved. These "facts of life" remove some of the incentive for suggesting an attempt of this nature. However, if a case can be made for an improvement in the overall military posture through the adoption of a suggested change, then there is a definite probability of its adoption. It is assumed that these institutional constraints can be overcome. This follows because if, "on the other hand, we can show Mr. Brown who operates a trucking company that he will make more profit by using his trucks for four years instead of one, he will have to prove we are wrong or stand convicted of willful stupidity. When the test is dollars and cents, we can talk a common language."[11]

The model is based on the idea that a meaningful comparison between total cost of the system and replacement life should be made. Since the product with which we are dealing is the capability to provide transportation services, a unit of this capability is taken as the measure of effectiveness. The method of vehicle procurement will be significant. There are two methods, instantaneous (or batch) procurement of all requirements and time phased (or continuous) procurement. The model is developed on the batch procurement basis with later modifications to accommodate the continuous method.

The Problem

The problem then is to determine the least cost of a system which produces the minimum acceptable capability to provide transportation

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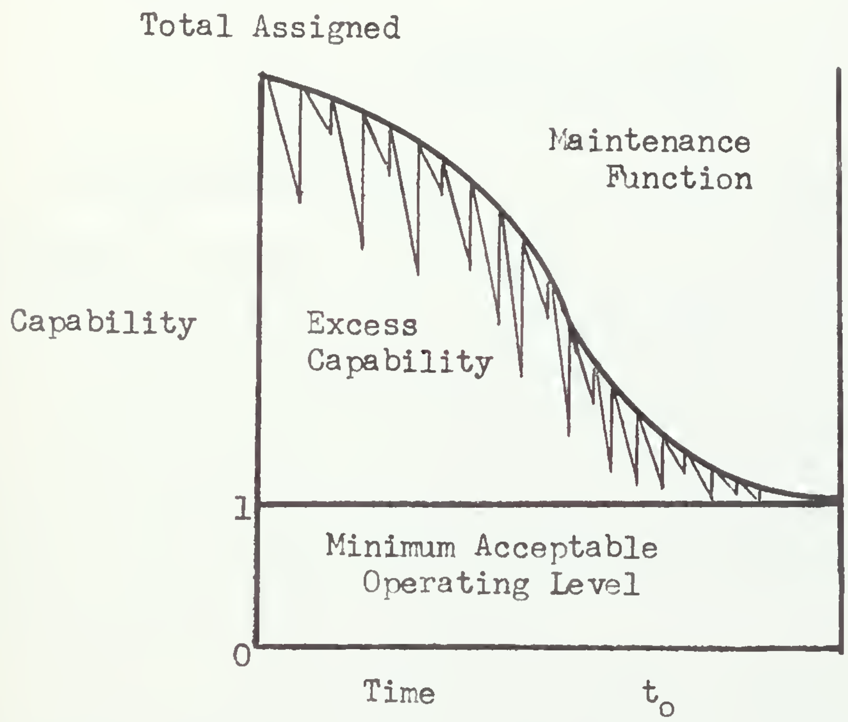
[11] George Terborgh, Dynamic Equipment Policy, (New York: McGraw-Hill, 1949), p. 14.

services, i.e., the minimum acceptable operations level. This minimum acceptable operating level is assumed to be constant. A graphical presentation of the situation follows which indicates that there is a series of discrete occurrences and associated maintenance actions which influence the amount of transportation available.



Excess capability would have to be purchased initially, or at a replacement interval, to ensure that the minimum acceptable operating level can be accommodated throughout the life cycle. This excess capability will be eroded by the demand generated by the deterioration due to age. During each discrete period, some maintenance failure occurs and there is a drop in the capability. The maintenance system responds in an attempt to restore the capability, but the restoration does not bring the total back to the original level. The decrease in the capability would represent the deterioration of effectiveness generated by the increasing age of the population of equipment.[12] The least cost

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[12] David A. Schrady, "A Deterministic Inventory Model For Repairable Items", <u>Naval Research Logistics Quarterly</u>, (Vol. 14, Sept. 1967), p. 391.

solution would be the period denoted by the time, t_o, which would minimize the average cost per unit time of the operations and maintenance costs and the acquisition costs of the total replacement quantity. This situation would appear to fall into the framework of a classic inventory problem of the order-level-lot-size system.

Inventory Approach

In replacement theory, a time is reached where the marginal cost of extending the life of an asset by one additional period exceeds the average cost per period of a replacement.[13] The costs which one would like to minimize for this system would be the costs of labor, repair parts, and working space plus the cost of purchasing the replacements. The cost of the replacement inventory is important because replacement decisions deal with the future.[14]

Vernon L. Smith states in his article "Economic Equipment Policies: An Evaluation" that:

> "For optimal replacement decisions we require knowledge of cost conditions only. An optimal pure replacement policy can be thought of as one which allows the firm to operate along the lowest possible cost-output curve."[15]

In the military application, the same relationship holds. The only significant difference is that we will hold the minimum output constant and find the least cost solution that assures at least this output.

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[13] George Terborgh, Dynamic Equipment Policy, (New York: McGraw-Hill, 1949), p. 201.

[14] Ibid., p. 38.

[15] Vernon L. Smith, "Economic Equipment Policies: An Evaluation", Management Science, (Vol. 4, 1958), p. 20.

We will be analyzing the problem in terms of the least money cost to the Department of Defense rather than social costs to the economy. Harry J. Gilman makes the case for using social costs:

> "Should resource allocations in defense establishment be subject to constraints that will assume minimum cost production? The answer to this question is yes, provided that it is social, not the Department of Defense, cost that is minimized. In the case of labor, for example, the cost that is relevant for defense production decisions is the cost that is necessary to attract a completely voluntary force."[16]

and

> "Moreover, with some exceptions, minimum cost production is socially desirable because it assures relatively less use of scarce (costly) resources."[17]

The question of social cost versus Department of Defense costs is surely a fertile field for research. However, since the purpose of this paper is to seek an optimal replacement model mainly within the current institutional and organizational framework, the distinction between social cost and Department of Defense costs will be ignored. The paper will be presented as the optimum allocation of resources made available for defense.[18]

The average replacement cost per period is simply the acquisition cost of the equipment divided by the number of periods of service. The marginal cost of extending the life of an asset for an additional period is a more complex problem. The amount of capability required to satisfy the minimum acceptable operating level changes as a

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[16] Harry J. Gilman, "Military Manpower Utilization", Defense Management, Stephen Enke, ed., (Englewood Cliffs: Prentice Hall, 1967), p. 248.

[17] Ibid., p. 248.

[18] Alain C. Enthoven, "Choosing Strategies and Selection of Weapons Systems", U. S. Naval Institute Proceedings, (Jan. 1964), p. 152.

function of service life due to the deterioration effect. Additional capability requires that there be vehicles assigned to provide that capability. This in turn requires that more personnel be assigned to operate and maintain this equipment. The amount of repair parts will vary with both the quantity of vehicles assigned and with the time (age) of the equipment. Summing these costs over the period in question will provide the marginal cost for extending the service life of the equipment for comparison with the average cost of the replacement. The point in time where these two costs are equal will define the replacement cycle, wearout point, or service life of the equipment.[19]

There are definite similarities between this problem and the normal inventory theory. Inventory theory can be modified to provide a methodology for selecting the optimal service life if values can be ascribed to time and capability. Inventory theory sums the cost of carrying the inventory, the cost of any shortages, and the cost of replenishment as an average total cost per period of time and then minimizes this sum.[20] In order to use this methodology, it is necessary to modify certain terms from their normal usages. Normally, the cost of carrying the inventory is concerned solely with storage, stocking, and issuing costs associated with the stock and the opportunity cost of the inventory value. Replenishment costs are concerned with the

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[19] Virginia W. Perry and others, "A replacement Requirement Methodology for Procurement of Army Equipment", (Vol. I, Fort Lee, Va., U. S. Army Logistics Management Center, July 1967) p. 4.

[20] An excellent treatment of the subject of inventory systems is available in Eliezer Naddor's Inventory Systems, (New York: Wiley & Sons, 1966).

administrative costs of the procurement or production effort. Shortage costs are more difficult to quantify, but include such items as lost sales, loss of good will, overtime payments, and special administrative efforts. It is clearly evident that these normal definitions do not fit the situation at hand. However, there are certain similarities which allows the theory to be altered to fit the circumstances.

First recall that our purpose is to develop a model which relates capability and cost and to seek an optimal solution in terms of age or service life for replacement of the vehicles that produce the capability. The capability itself does not have a directly quantifiable price. Therefore, if we consider carrying a stock of capability and if we replenish capability, the case more nearly fits the terms of normal inventory theory. As is possible in inventory theory, we can choose to adopt a no-shortage type system. When we specify that the capability must equal or exceed the minimum acceptable operating level, in effect we are stating that there can be no shortage.[21] The cost of carrying the inventory is the cost of maintaining the stock of capability. This is the cost of operating and maintaining the stock of vehicles which produces the capability, i.e., the cost of operators, maintenance personnel, and repair parts. When vehicles reach the replacement age, the replenishment cost is the cost of acquiring new vehicles.

Since our inventory stock is capability, the demand for this stock

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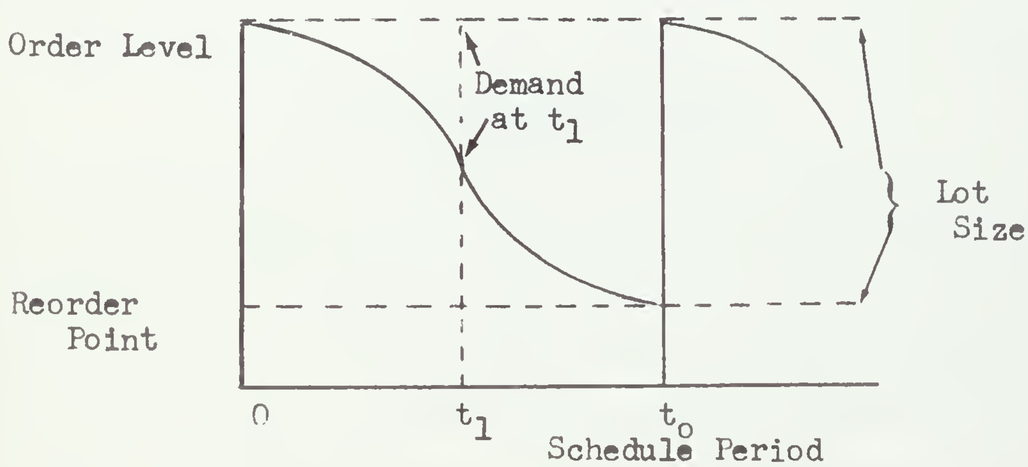
[21]Here we should digress to point out that if the decision maker could, or can, place a meaningful value on a failure to maintain the minimum acceptable operating level, the theory and model can be easily changed to accommodate this situation. But the task of placing a value on such an occasion seems to be the same as answering the question, "How much is defense worth?" The only difference is the matter of degree and relative importance.

is the erosion caused by deterioration due to age. The demand is represented by the percentage of vehicles out-of-service. This increases with time and, therefore, produces less capability. If this demand is known, the system can be treated as a deterministic inventory system. However, should individuals who desire to use this approach prefer to consider a probabilistic system, inventory theory can accommodate such a change. The inventory system which most nearly fits our system is the deterministic order-level-lot-size system. This system solves for the proper order level, which is the number of vehicles required in the inventory to produce the capability over the time period. It will also solve for the scheduling period, which is the economic service life in this case. The order point is the level at which it is necessary to reorder a lot size to regain the order level.[22] The following diagram shows these relationships.



These relationships are optimized to minimize the applicable costs. The system requires that carrying cost and the replenishment cost be computed on the basis of the average cost per unit time. These costs are commonly assumed to be constant. However, the theory can be modified to include costs which vary with time. In either case, the desired result

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[22] Eliezer Naddor, Inventory Systems, (New York: Wiley and Sons, 1966), p. 78ff.

is the time t_o which minimizes the average cost per period of service. This is the definition used by William T. Morris for the economic service life of an asset.[23]

By using the modified inventory theory as a guide for our model, we can develop a model which defines the least cost solution for the system as a whole in terms of the economic service life, subject to the maintenance of a minimum acceptable operating level. This has the same effect as that which would be obtained if we could measure in dollars the benefits of capability and relate this to vehicle age. In the model we normalize the minimum acceptable operating level. In other words, we provide each unit that is assigned these vehicles the necessary capability to perform its mission throughout the entire period. In the initial stages, these units have more capability than is required, but this decreases with time until the capability just equals the requirement at the replacement time.

Deterioration Due to Age

A principal factor in determining life expectancy of a tactical military vehicle is attrition. There are two separate types of attrition which must be considered, casualty or loss (mortality) and the deterioration due to age. Mortality attrition is assumed to occur at a constant rate. These losses are replaced as they occur. Accordingly, this feature of the model will be considered later along with other refinements to the basic model.

The other type of attrition, deterioration due to age or use, can be handled so simply as mortality. First, it is apparent that if

[23] William T. Morris, Analysis of Management Decisions, (Homewood, Ill.: Richard D. Irvin, 1964), p. 194.

the quantity of equipment is maintained at a constant or fixed level (i.e., the TO&E allowance) and there is deterioration suffered with age, there will be a steady decline in the effectiveness of the unit. To maintain a minimum acceptable operating level requires that additional units of equipment be added as the deterioration is experienced or that additional units be assigned initially and allow the deterioration to erode the excess until the capability equals the operating requirement. When this occurs, it is time for replacement. The former alternative poses one of the institutional and organizational constraints of the military situation which must be applied in the development of the model. A continually changing TO&E and personnel allowance for each unit according to the age distribution of the assigned equipment poses a serious problem for the logistics planners. It would seem to be a hopeless endeavor to formulate a model predicated on this approach. This alternative is, therefore, discarded as unfeasible and a solution is sought with the other alternative. This is consistant with the use of the inventory approach.[24]

In the military, where the support organization must be provided for an expected or normal situation, we will proceed on the basis of adapting the analysis to the existing institutional and organizational constraints and seek an optimal solution in this environment. Since the

[24] In an application with a more elastic support response, such as purchase of the required support from the civilian economy, the alternative of adding additional units as the deterioration occurs should be investigated. Here, the costs associated with excess capability would not be encountered as they are in the military application. If, on the other hand, the user must provide his own support establishment, he can vary the size and tempo of the operation by hiring additional men as they are required. This will allow the total costs to be reduced and will provide an optimal solution different from that expected under a situation where the costs will be constant throughout the period.

variable manpower and support concept can not be used, the solution must be of the type which provides excess capability in the early periods. We, then, wish to minimize the cost of acquiring the replacements for the inventory of vehicles required and the cost of operating and maintaining them throughout the service life, subject to maintaining the minimum acceptable operating level at all times.

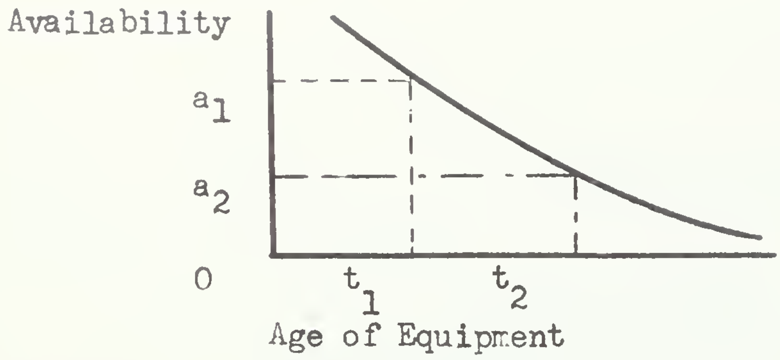
The Maintenance Function

In order to develop our model along the lines of the inventory approach, it is necessary to identify the demand that will erode the excess capability. This demand is generated by the interaction of routine maintenance requirements, the probability of failure, and the ability of the maintenance support organization to restore any out-of-service equipment to use. This demand is represented by a percentage of vehicles out-of-service with time. A short discussion of some of the relationships in the production function is necessary for a better understanding of the development of the model.

If the problem is viewed as one where a production function

$$Q_a(t) = t'Q_T^u L^v R^w S^x \qquad (u+v+w+x = 1)$$

where $Q_a(t)$ is the quantity of vehicles available at time t (capability), t' is the adjustment for the deterioration due to age, Q_T is the quantity of vehicles assigned, L is the number of personnel assigned, R is the amount of repair parts, and S is the working space. If we assume that the factors, Q_T, L, R, and S, are fixed in any mix, then the function will only vary with t'. Since t' will decrease with age, as the deterioration increases, the following curve will represent the relationship.



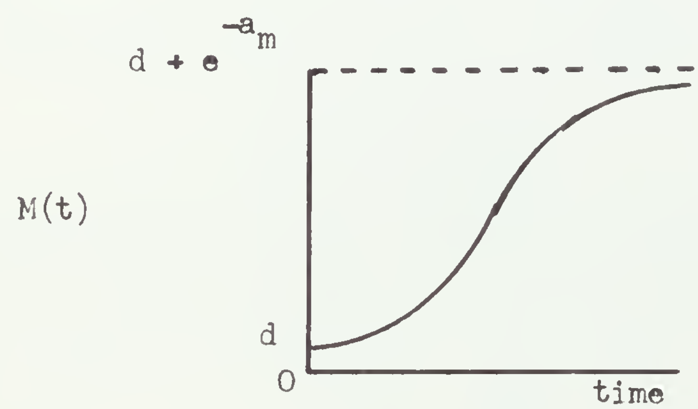
Due to the institutional and organizational framework of the military, the mix of the factors in the production function will be fixed for the service life of the equipment. Under this constraint, an increase in availability can only be obtained at a corresponding reduction in the age of the equipment. However, the initial composition of the mix can be established through the operation of a cost equation and the total quantity of vehicles required. This is possible because of some embedded relationships. These embedded relationships are developed along with the model. At this stage, we are able to see that optimizing availability and time can produce the solution to the replacement model.

Availability is a proxy for capability, for if a vehicle is available it can produce a vehicle unit of capability. Using the availability of vehicles, we can develop a function with time which will represent the amount of capability at any given time. In the consideration of this feature, there are two major items. of interest. There are the routine service requirements which must be performed regardless of the age of the equipment. The acceptance checks, weekly, quarterly, and annual inspections (or mileage interval checks) reduce the availability of the vehicles. Maintenance failure, which follows some probabilistic form, is the other. Since these vehicles are actually complex equipment

consisting of many components, we can expect that the function will be a logarithmic, reciprocal transformation function with time which will be asymptotic to some value. We can then hypothesize that the maintenance function is of the form[25]

$$M(t) = d + e^{-(a_m + b_m/t)} \qquad (1)$$

where d is a constant established by the preventative maintenance requirements, $e^{-(a_m + b_m/t)}$ is the exponential characterization of the maintenance malfunctions, and M(t) is the portion of the inventory of vehicles that can not be utilized. This function is shaped as shown in this sketch.



This form can be rationalized on the basis that new vehicles do not require much maintenance at first, then failures begin to increase as the units age until the time that the age of the components begin to stabilize. At this time, it begins to increase at a decreasing rate until it assumes an almost uniform rate, the asymptotic value. M(t) would then operate as a percentage of the total vehicles assigned.

When the total available capability reaches the minimum acceptable operating level, the capability must be replaced. Therefore, if we let S' denote the number of trucks required at the beginning of a

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[25]R. W. Drinkwater and N. A. J. Hastings, "An Economic Replacement Model", Operational Research Quarterly, (June 1967), p. 20.

replacement cycle for every available truck at the end of the cycle, we have

$$S' - M(t_o)S' = 1 \tag{2}$$

where t_o is the replacement time, i.e., the length of a replacement cycle. Then by substituting (1) into (2) and solving for S', we find that

$$S' = \frac{1}{1 - d - e^{-a_m - b_m/t_o}} \tag{3}$$

This relationship shows that even at time zero we would require more than one unit of capability to be assigned to produce the minimum acceptable operating level of one unit of capability. As the replacement time t_o is increased, a larger amount of assigned capability is required.

The Cost Function

We can use this variable capability with time in the following cost equation and find the optimum or least cost replacement cycle time.

$$C = c_1S' + \frac{c_3S'}{t_o} \tag{4}$$

where C is the average total cost per unit time, c_1 is the average cost per unit time per unit of assigned capability for operation and maintenance, and c_3 is the acquisition cost of each replacement item. The first term is the operating cost term and the second the replenishment term. The replenishment term gives the average cost of the replenishment per unit time. If we minimize this expression with respect to time, we will have the solution value we seek. But embedded in c_1 are some subordinate relationships which must be made explicit before we can attempt to minimize the expression.

In the development of the maintenance function, we noted that

there were three factors that contributed to the ability to restore maintenance malfunctions. These were labor, repair parts, and working space. Labor and working space are a function of the quantity of vehicles while repair parts is a function of both the quantity of vehicles and time. In order to compute the amount of direct labor required, the quantity and type of vehicle is needed. Each type of vehicle is graded with an appropriate factor to denote its vehicle equivalent and the criteria manual specifies the number of personnel required to support a vehicle equivalent. Therefore, there will be a set number of personnel per vehicle and this number will vary with the total number of vehicles assigned, Q_T. A linear relationship between L and Q_T closely approximates the criteria manual data

$$L = a_1 + b_1 Q_T \tag{5}$$

dividing this expression by Q_T will provide the number of personnel for each vehicle

$$\frac{L}{Q_T} = \frac{a_1}{Q_T} + b_1 \tag{6}$$

Note that no adjustment is provided for the age of the equipment.[26] Regardless of this shortcoming, this relationship is quite important to the analysis because of the proportion of cost represented by direct labor. This relationship gives the total number of personnel required for the direct labor. Indirect support personnel, such as unit administrative, medical, housekeeping, and the like, must also be included. The U. S. Army Logistics Management Center has found that there is a direct relationship between the number of direct labor hours required

[26]No evidence could be discovered which would allow this relationship to be stated in terms of time or age of the equipment. It is reasonable to assume that some average age for the vehicles to be supported has been used in computing the criteria requirements; however, it appears that this has been an implicit consideration developed through the years.

and the indirect supervision and support costs. Also, it found that the cost of utilities and repair to working space as well as amortization of the tools, equipment, and supplies are directly related to the direct labor hours. It derives a simple inflator of the cost per direct labor hour that closely approximates the total cost for these items.[27] Therefore, the price of a unit of labor is inflated by an appropriate amount to produce the cost per unit of labor (P) which is used in the model. This will give the following relationship

$$L' = P\left(\frac{a_1}{Q_T} + b_1\right) \tag{7}$$

where L' is the cost per unit time per unit vehicle, P is the inflated cost per man per unit time. Since the inflator included the amortization for the working space and tools, the costs of these items need not be stated separately.

The cost of repair parts varies with time due to the increasing failures. We can assume that the cost function will be some exponential form similar to that of the maintenance function. However, it is normal to have a high limit for the repairs to any one vehicle. This limit is set by the repair/scrap policy. The repair/scrap policy is normally stated in terms of a portion of the cost of the vehicle (i.e., 65 percent of the acquisition cost). Making these assumptions, the repair parts function is of the following form

$$r(t) = e^{a_r - b_r/t} \tag{8}$$

where r(t) is the cost of the repair parts at time t. As with the cost of personnel, there are similar embedded costs associated with repair

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[27]Virginia W. Perry and others, "A Replacement Requirement Methodology for Procurement of Army Equipment", (Vol. I), U. S. Army Logistics Management Center, Fort Lee, Va., July 1967, p. 9.

parts. The cost to stock, store, and issue the repair parts is properly one of the costs of maintenance. By inflating the repair parts cost by an appropriate amount we can include these embedded costs.[28] This inflator (R) is added to the repair parts function as a multiplier and results in the following

$$R(t) = Re^{a_r - b_r/t} \tag{9}$$

To determine the average cost per unit time (c_1) we must integrate (9) over the interval from zero to time t_o and divide the result by t_o and add (7). The function will then be

$$c_1 = P\left(\frac{a_1}{Q_T} + b_1\right) + \frac{R}{t_o}\int_0^{t_o} e^{a_r - b_r/t}\,dt \tag{10}$$

The integral term is not an easy one to evaluate; however, with the use of the following substitutions an analytical expression can be obtained. First factoring out all terms that do not vary with t

$$\frac{R}{t_o}\int_0^{t_o} e^{a_r - b_r/t}\,dt = \frac{Re^{a_r}}{t_o}\int_0^{t_o} e^{-b_r/t}\,dt$$

Then letting

$$u = \frac{t_o}{t};\ t = \frac{t_o}{u};\ dt = -\frac{t_o}{u^2}du$$

and substituting, we have

$$\frac{Re^{a_r}}{t_o}\int_{\infty}^{1} \frac{-e^{-(b_r/t_o)u}\,t_o\,du}{u^2}$$

this expression is the same as

$$Re^{a_r}E_2(b_r/t_o)$$

where

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[28] Ibid., p. 9ff.

$$E_n(z) = \int_1^{\infty} \frac{e^{-zt}}{t^n}\,dt$$

with z equal to b_r/t_o and n equal to 2. This form is shown in the Handbook of Mathematical Functions to have an analytical approximation as follows:[29]

$$E_n(z) \cong \frac{e^{-z}}{z+n}\left[1 + \frac{n}{(z+n)^2} + \frac{n(n-2z)}{(z+n)^4} + \frac{n(6z^2 - 8nz + n^2)}{(z+n)^6}\right] \quad (11)$$

Substituting (11) into (10) and (10) into (4) will give the following cost equation

$$C = (P' + R')S' + c_3S'(t_o)^{-1} \quad (12)$$

where $P' = P\left(\frac{a_1}{Q_T} + b_1\right)$

$$R' \cong t_o Re \frac{a_r - b_r/t_o}{b_r + 2t_o}\left[1 + \frac{2t_o^2}{(b_r+2t_o)^2} + \frac{4t_o^3(t_o - b_r)}{(b_r+2t_o)^4}\right.$$

$$\left. + \frac{4t_o^4(3b_r^2 - 8b_rt_o + 2t_o^2)}{(b_r+2t_o)^6}\right]$$

$S' = (1 - d - e^{-a_m - b_m/t_o})^{-1}$

c_3 = acquisition value of the replacement

t_o = replacement cycle time (economic service life)

The Model

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[29] U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55, Handbook of Mathematical Functions. Milton Abramowitz and Irene A. Stegun, eds., (Washington: U. S. Government Printing Office, 1964), P. 228ff. This book provides a table of values for some values of n and z. There is an error term contained in the analytical representation that has been ignored.

[30] William T. Morris, Analysis of Management Decisions, (Homewood, Ill.: Richard D. Irvin, 1964), p. 194.

In the normal course of an analytical solution to this equation, we would take the first derivative of the average total cost function with respect to t_o, set it equal to zero and use the solution value of t_o as the economic service life or replacement cycle time. The second derivative would be evaluated to ascertain that the solution was in fact a minimum extremum. However, due to the complexity of the equation, iterative computer techniques will be used which will serve the same purpose. The model will give the average total cost of the operations and maintenance costs and the acquisition cost of the replacements per unit time. The minimum point on this average total cost curve will denote the economic service life of the equipment.[30] The model will solve for the replacement cycle time t_o. With this value the required quantity of vehicles to be assigned can be computed by the use of equation (3). With the quantity of vehicles known, the personnel requirements can be computed. Knowing these requirements, the logistics planners can produce the TO&E's and Personnel Allowances needed for each unit. The model will accommodate itself to the institutional and organizational framework of the military since these modifications will only be required periodically at the replacement cycle time.

Summary

In the development of the model, it was apparent that determining the optimal mix between the factors, availability and time, would be a

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[30] William T. Morris, Analysis of Management Decisions, (Homewood, Ill.: Richard D. Irvin , 1964), p. 194.

difficult process as neither of these factors has a quantifiable price. By the use of an inventory approach and the definition of the subordinate functional relationships, it has been possible to impute implicitly prices for these factors. Through this indirect pricing method, it is possible to seek an optimal mix between availability and time. This is accomplished by a substitution of newer vehicles for labor and spare parts. A simultaneous solution of the total capability required and the economic service life is produced by the interaction of the marginal productivities. Indirectly, then, the substitution is a capital-labor substitution of the normal type. With the proper functional relationships to produce the parameters, the model can be used to select the proper number of vehicles required to satisfy the normalized minimum acceptable operating level at the economic service life of the equipment. The maintenance function determines the amount of the assigned capability which is available at any time by operating as a percentage of the total capability required. Using this relationship in the total cost equation allows for indirect pricing and substitution of factors to select the economic service life. The cost function operates to increase the cost of operations of a unit of capability along with the decrease in effectiveness due to deterioration because of age.

This relationship could be shown explicitly through the use of a Lagrange constraint equation with the production function and the cost of the factors of the production. However, this relationship would require that the production function be known in its exact form. Since the exponents of the assigned vehicles, labor, repair parts, and space for the production function are not known and data for their

derivation are not available, the inventory approach was chosen.

The model computes the average or expected cost per unit time for the whole system, including the initial requirements. The solution value for the model is the minimum point on the average total cost curve with respect to economic service life. The solution of the model provides the information necessary to compute the total assignment needed to insure that the minimum acceptable operating level is maintained throughout the service life of the equipment

The model is formulated with continuous functions despite the discrete nature of some of the values. The model could have been formulated in the discrete mode, but the corresponding complexity of the simultaneous solution of the subordinate relationships militated against this approach. Understanding this possible shortcoming of the model, the results are superior to those presently in use that do not consider the system in its entirety. The model does not accept the thesis that the military requirement is a fixed number of vehicles. Rather, the model states that for any given requirement stated in terms of the desired capability there is a trade-off between the availability and the age of the equipment. This trade-off is optimized and the results are stated as the replacement cycle year or economic service life of the equipment.

In order to complete the analysis, the basic model must be expanded to include the continuous procurement situation, mortality attrition, salvage value, and discounting to present value. These features will be treated in the following chapter.

CHAPTER III

ELABORATION OF THE MODEL

> "Operations research and, to an even greater degree systems analysis seem to be more nearly engineering than science. For the purpose of making a distinction here, one might say that science finds things out while engineering uses the results of science to do things cheaply and well." 31

Other Considerations

The basic model would only be valid for an extremely limited application because it does not consider certain influences. The situation where there is continuous procurement, rather than batch procurement of equipment, must be considered along with effects of attrition, delayed expenditures, and resale or salvage value. The basic model is expanded to include these considerations.

Continuous Procurement

In the basic model, both the maintenanace and repair parts functions were developed on the basis of an initial procurement of all the equipment required during the expected life of the item. This type of procurement was labeled "batch procurement" to distinguish it from procurement at some uniform rate throughout the life of the equipment. Batch procurement allowed all the items to age together. However, if

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[31] James M. Henderson and Richard E. Quandt, Microeconomic Theory, (New York: McGraw-Hill, 1958), p. 291.

the equipment is purchased at some uniform rate throughout the life of the item, at any given time there would be vehicles on hand with varied ages. For simplicity, we assume that the equipment is purchased in equal periodic increments throughout the economic service life of the item. This produces a mixing of the age of the vehicles similar to that noted by McCloskey and Coppinger for the components.[32] When the steady state is reached, both the maintenance and repair parts functions must be modified to reflect the average age of the vehicles at time t. If we take the sum of the function values of the various ages and divide by the number of vehicles, we will have a value for the average age at time t.

$$M(t_o) = d + \frac{1}{t_o} \sum_{i=1}^{t_o} e^{-(a_m + b_m/t)}$$

and the repair parts function would be the sum of the function values of the integral divided by the number of vehicles.

$$R(t_o) = \frac{R}{t_o} \sum_{i=1}^{t_o} \frac{1}{i} \int_o^i e^{a_r - b_r/t} dt$$

These values are predicated on equal amounts of procurement during each period. These values can be approximated in continuous terms by substituting integration for the summation.

$$M(t_o) = d + \frac{1}{t_o} \int_1^{t_o} e^{-(a_m + b_m/t)} dt \qquad (13)$$

and

$$R(t_o) = \frac{R}{t_o} \int_1^{t_o} \frac{1}{x} \int_o^x e^{a_r - b_r/t} dtdx \qquad (14)$$

[32]Joseph F. McCloskey and John M. Coppinger, Operations Research for Management, (Baltimore: Johns Hopkins Press, 1956), p. 336.

Substituting (13) for (1) and (14) for (9) will modify the basic model to accommodate the situation where there is continuous procurement. A solution of the model with these substitutions will indicate the economic service life under the steady state of equal annual increment procurement. In the situation of transition from a previous policy of batch procurement to continuous procurement, neither model can be used. However, if the transition is made by replacing the appropriate quantity of vehicles each year in accordance with the continuous policy, a steady state can be reached in t_o years. During the transition, there will be a continually changing requirement for both the number of vehicles and personnel to support them. This situation does not appear to be of sufficient importance to require the development of a model to satisfy the efficient allocation during this limited period. Accordingly, the formulation of a model for the transition period will not be provided in this paper.

Attrition

There are two types of attrition, deterioration due to age and casualty or loss (mortality). In the previous chapter, we treated the effects of deterioration due to age by the inclusion of the maintenance function. Mortality attrition requires a different treatment. Although mortality attrition will usually follow some exponential decay, if the mortality replacements are made in each period, so that the quantity on hand remains a constant, the replacement requirements will stablize at some constant rate each year.[33] This was the same conclusion reached by

[33]Ruel V. Churchill gives a detailed and complete treatment of the subject of attrition in his book Operational Mathematics, Second Edition, (New York: McGraw-Hill, 1958), p. 65ff.

the Army Logistics Management Study, but they also found that the data presently collected were not of sufficient detail to identify the rates accurately. One of the recommendations of that study was to modify the reporting procedures to allow these rates to be identified. I have encountered the same difficulty in ascertaining the appropriate rates. In the interim, until the data will produce the correct rates, a subjective judgement will be required. For the example, I have chosen an average rate of 5 percent per year for the 5 ton truck, M 52, for the world-wide (or total inventory) rate.

In the batch procurement system, sufficient replacements must be purchased for the entire period with the excess being held in the supply stocks until they are needed. This will increase the replenishment cost (c_3) by $.05t_o$. There will also be an increase in the cost of these items by the amount necessary to stock, store, and issue these replacements. These relatively high cost items will have a lower SSI factor than that associated with the repair parts. Again a subjective judgement will be required until the appropriate factors are available. For the example, a factor of 1.2 has been selected as an appropriate cost inflator. Multiplying the replacement cost by this inflator and adding it to the replenishment term will produce the following

$$\frac{c_3(1 + .06t_o)}{t_o(1 - d - e^{-(a_m + b_m/t_o)})} \tag{15}$$

as the second term of equation (12).

In the continuous case, the mortality replacements can be purchased during each period and there is no need to incur the SSI costs. The modification of the replenishment term will then be provided by multiplying the replacement cost by the attrition rate and adding it to the

replenishment term. This will produce the following

$$\frac{c_3(1 + .05t_o)}{t_o(1 - d - \frac{e^{-a_m}}{t_o}\int_1^{t_o} e^{-b_m/t}\,dt)} \tag{16}$$

Substituting these terms into the cost equation (12) for the appropriate situation will provide the adjustment for the mortality attrition. A corollary subject which could be considered is technological obsolescence of the equipment. In the case of vehicles, this feature has been ignored due to the minimal influence it plays in these particular items. If it were desired to include such a consideration, it could be introduced into the model in a similar manner. Vernon L. Smith offers an obsolescence technique in his article "Economic Equipment Policies: An Evaluation" which could be used.[34] However, the technique requires that the value of the benefits be identified. Most military missions do not lend themselves to identification of the value of the benefits. Also, improvements in the ability of the vehicle to perform would probably be submerged in the requirement for a given number of vehicles required to perform any given mission. Accordingly, the subject of technological obsolescence has been ignored.

Present Value

When there is a cost stream which spans several years, the question of discounting to a present value comparison is raised. In the military case, where benefits can not be quantified by a price, a present value comparison of the benefits to costs can not be accomplished. Mr. Ted Rathbun has made the following statement relative to this problem to

[34]Vernon L. Smith, "Economic Equipment Policies: An Evaluation", Management Science, (Vol. 4, 1958), p. 22f.

the Congress:

> ". . . I would like also to say, and this is pointed out in the report that there are undoubtedly cases where the benefits can't be quantified, but even here it would seem helpful to use a reasonable discount rate and present to the decision maker the present value of the costs of these two programs and let him use his judgement in placing a value on these hard to quantify benefits."[35]

As long as there are alternative uses for the money which will be, or may be, spent by the Department of Defense, it would seem to be proper that these alternative uses be considered. Costs or savings associated with any expenditure must be carefully weighed in this light. the delay of a replacement expenditure by one year will produce added costs of maintenance of a certain amount. The decision relative to the delay should be made after an evaluation of the possible savings that the delay may produce or of the possible increased costs. Discounting takes into account the alternative use to which the funds could be devoted and subtracts this amount from the total to arrive at a present value. For example, if a one hundred dollar expenditure is being considered to be made now or one year from now, the true costs of these two expenditures are different. The difference is the amount that the one hundred dollars could earn during the year. If the rate of return on invested funds was 10 percent, the present value cost comparison would be one hundred dollars for $100 spent now and ninety dollars for $100 spent one year from now. However, if the delay will incur added expenses of fifteen dollars spread evenly over the year, then the present value of the delayed cost would be $103.57. From this simple example, it is apparent that even where costs alone are considered, rather than a cost-benefit comparison, the role of discounting can be significant.

[35]U. S. Congress, Subcommittee on Economy in Government of the Joint Economic Committee, Hearings, "Interest Rate Guidelines for Federal Decisionmaking", U. S. Govt, Printing Office, 29Jan68, p. 14.

If we accept that the discounting technique is a valuable tool when used in decisions which span a period of time, the next question that arises is what is the proper discount rate to be used? For the usual profit maximizing entrepreneur, it is usually the cost of borrowing the funds that is the controlling feature. For the government, it is not quite so simple. The cost of borrowing long term money is currently over five percent. However, when the government borrows money, it removes this money from the private sector where it would have produced certain revenues. Corporate and personal taxes on the amount borrowed are foregone when these funds are removed from the private sector. Different Government agencies have derived different rates which range from three to twelve percent.[36] The Department of Defense uses an opportunity cost rate of ten percent in its calculations.[37] Although, there is still much controversy as to the proper rate which should be used, most economists are agreed that the proper rate falls in the range of from seven to fifteen percent. In this treatment, a discount rate of ten percent will be used. The cost equation will be multiplied by the continuous discount term e^{-kt_o}, where k equals the discount rate chosen (i. e., for the example, $e^{-.1t_o}$).[38] This treatment is chosen even though it will give an effective rate of slightly more than ten percent.

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[36]Comptroller General of the United States report to the Joint Economic Committee. Contained in Hearings, Ibid., p. 34.

[37]Department of the Army, "Utilization of Military Resources", T. Arthur Smith and Ogden O. Allsbrook, eds., 1967.

[38]The model could have been formulated in the discrete mode. An excellent discrete model with simple discounting is contained in William T. Morris', The Analysis of Management, p. 199ff. Vernon L. Smith's Investment and Production gives a clear and concise esposition of Hotelling's "fundamental formula" with continuous discounting at a constant rate of interest and G. A. D. Prienriech's "aggregate goodwill" method of continuous discounting. On page 20 of his article "Economic Equipment Policies: An Evaluation", Smith presents his continuous discounting method.

Salvage Value

There would seem to be but one item remaining to complete the model. This is consideration of the salvage or resale value of the items being replaced.[39] Normally military vehicles do not enjoy a vigorous second-hand market, primarily because most tactical military vehicles have been held in the active inventory until they had only salvage value for the metals they contain. If the model should indicate that these vehicles should be replaced more often, it is possible that a viable second-hand market could be generated. However, it is more probable that this situation would tend to have its greatest impact in an area where prices are extremely difficult to quantify.

At present, many of the tactical military vehicles that have been replaced have found an outlet through the Military Aid Program (MAP). One of the criticisms of this program from our allies has been that they have not been able to acquire sufficient repair parts, especially when there has been a technological replacement in the U. S. active inventory. A system which would make more vehicles available, while the U. S. forces were still using the same type, would tend to eliminate some of the repair parts problem. But more important, can we rationally justify the use of second-hand items by our allies? The answer to this question can be given with the model with the appropriate parameters modified to accommodate the cost of labor and repair parts to the recipient MAP country. In an area where the wage rates were substantially lower, an item would have an economic service life remaining even though it had reached

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[39] Dale W. Jorgenson and others, Optimal Replacement Policy, (Rand Corporation, Amsterdam: North Holland Press, 1969), p. 6.

the economic service life under the situation in the U. S. forces. Clearly, the alternative use of these vehicles, after they have reached their economic service life in the U. S. environment, can be meaningful and supported by analysis conducted with the model. Identifying the benefits associated with the alternative use in the MAP program in order to include an appropriate measure in the model would itself be a subject for a thesis. Since there is no history available for the establishment of resale values for tactical military vehicles nor are there identifyable MAP values that can be used, some other approach must be used to develop a measure of the salvage or resale value.

Vernon L. Smith states that:

> "A common view that one finds in both the theoretical and the trade literature is that units of industrial equipment, particularly trucks, tend to fall in value in their first year by a constant proportion of their new price, and then to decline in value at a constant percentage rate per year thereafter.[40]

He further identified the appropriate equation for the resale value as follows:

$$\text{Resale} = c_3 \, (y_4 - y_5 t_o) \tag{17}$$

where c_3 is the price paid for the new item, y_4 is one minus the depreciation that the item suffers initially, and y_5 is the constant percentage rate per year. From an empirical analysis of the Blue Book values for a large cross country tractor, he found that the truck lost 38.5 percent initially and at a rate of 7.06 percent per year thereafter. By subtracting (17) from the replacement cost in (15) and (16) will give the following replenishment term

$$\frac{c_3(1 + y_3 t_o - y_4 + y_5 t_o)}{t_o(1 - d - z)} \tag{18}$$

[40] Vernon L. Smith, "Economic Equipment Policies: An Evaluation", Management Science, (Vol. 4, 1958), p. 28f.

where y_3 equals the attrition rate, y_4 equals one minus the proportion of cost lost initially, y_5 equals the constant rate per year, and z equals the exponential maintenance function.

Summary

The model has been expanded to allow a choice of either the batch or continuous procurement mode. Mortality attrition has been incorporated by the modification of the replenishment term, in either mode. Comparisons at present value were provided for by the inclusion of discounting at an acceptable rate. Finally, the salvage or resale value was introduced by a modification of the replenishment term. The resultant model is set forth as follows

$$C = S'e^{-kt_o}(P' + R' + c_3A'/t_o) \tag{19}$$

where

$$S' = (1 - d - y_1 e^{-a_m - b_m/t_o} - \frac{y_2}{t_o}\int_1^{t_o} e^{-a_m - b_m/t_o}dt)$$

$P' = Pa_1/x + Pb_1$ (x = total of all vehicles assigned to unit)

$$R' = y_1Rc_3\int_0^{t_o} e^{a_r - b_r/t_o}dt + \frac{y_2Rc_3}{t_o}\int_1^{t_o}\frac{1}{v}\int_0^{v} e^{a_r - b_r/t}dtdv$$

$A' = (1 + y_3t_o - y_4 + y_5t_o)$

k = interest rate or discount rate

y_1 = one for batch procurement, zero for continuous procurement

y_2 = zero for batch procurement, one for continuous procurement

y_3 = 1.2 times attrition rate for batch, attrition rate for continuous procurement

y_4 = one minus initial depreciation

y_5 = yearly depreciation rate

The basic definition of the derivative was used in the development

of a computer program to find a solution for the model.

$$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

The derivative was computed and a root finder subroutine used to find a solution value. This value was tested for a minimum by numerical comparison. The tested value was used as the solution value for the model.

CHAPTER IV

DEVELOPMENT OF THE PARAMETERS

> "One cannot obtain valid conclusions just by assuming laws of failure, however simple or appealing, without confirming these by actual observation. Neither is it safe to extrapolate failure rates far beyond the range of the observed data, nor, as we have seen is it safe to assume a component failure rate constant." [41]

General

In the foregoing chapters, a model has been developed which should find the solution value for the economic service life of a tactical military vehicle. Investigation has revealed that the available data do not readily produce the necessary relationships for exercising this model. Exact identification of the functional relationships is not necessary for a realistic demonstration of the model. Gross approximations of the parameters can be derived through mathematical manipulation. This is the approach that will be taken.

Personnel Function

The personnel function was assumed to be a linear function of the total quantity of vehicles assigned. It is intuitively reasonable that it will require more personnel to operate and maintain two trucks than is required to operate only one truck. This logic, alone, is not

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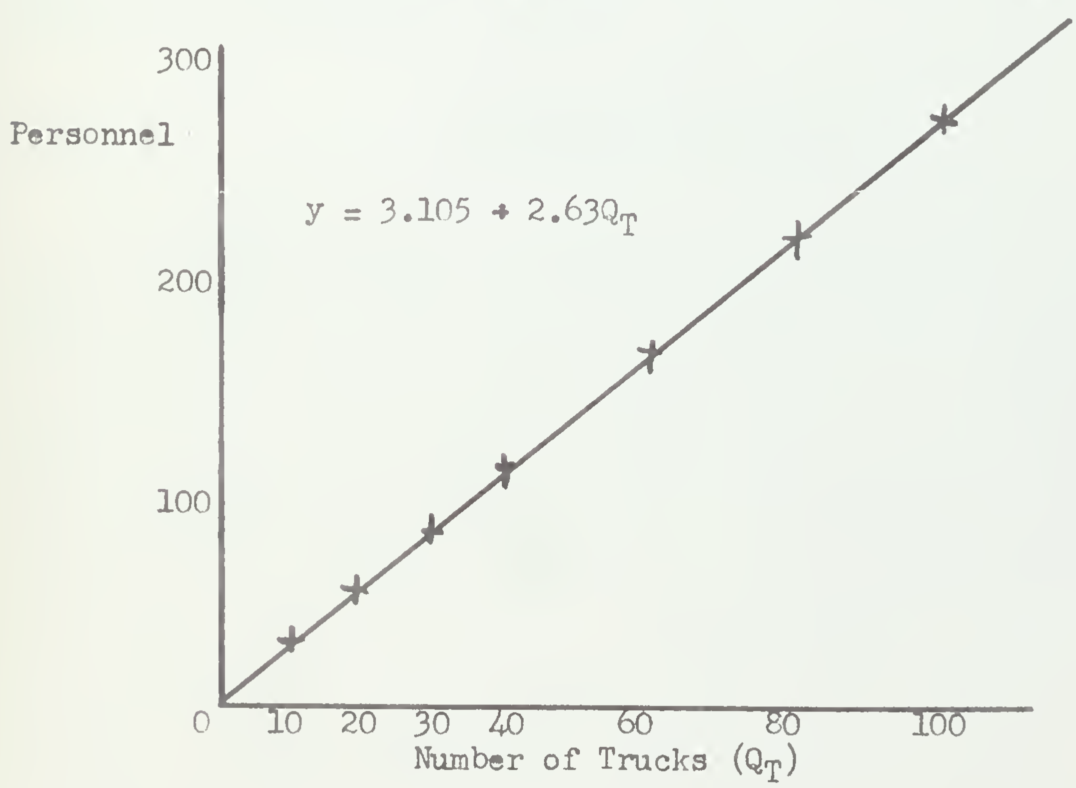
[41] Joseph F. McCloskey and John M. Coppinger, Operations Research for Management, (Baltimore: John Hopkins Press, 1956), p. 338.

sufficiently precise for our purposes. We must know within reasonable certainty the increase needed for each additional truck. By use of the criteria provided in the Criteria Manual it is possible to compute the number of maintenance and operations personnel required.[42] The manual is designed to formulate the requirements for a unit or organization which operates a group of vehicles. Since a unit may have several different types of vehicles, a vehicle equivalent method is used to provide the basis for the personnel requirements. Using the vehicle equivalent method, factors are provided for each type of mechanic and operator needed.

Discrete values can be determined for notional units of a given size. In order to provide an equation, in continuous terms, a least squares line is fitted to the plot of the notional values. A nominal unit of ten vehicles was selected and personnel requirements for the



[42]U. S. Marine Corps, Personnel Requirements Criteria Manual, Chapter 23. (5 August, 1964).

intervals were computed with the results as shown in the above graph. The computed values were as follows

Unit Size (vehicles)	Personnel Required
10	31
20	55
30	80
40	110
60	159
80	215
100	266

This least squares fit produced a variance of .022079. Therefore, the parameter values of $a_1 = 3.105$ and $b_1 = 2.63$ can be substituted in the model when using it for the M 52 type vehicle, in the general support mission. Other values for the parameters can be generated for other vehicles which have a different vehicle equivalent or vehicles that are operated in the organizational support role. For example an internal support vehicle of the one vehicle equivalent type will generate the values $a_1 = 1.552$ and $b_1 = 1.303$.

The Maintenance Function

The maintenance function posed a much more difficult problem in locating data from which to derive the coefficients. A mass of data is collected on vehicles; however, it is primarily intended for use in evaluating the supply response. Under the Army's Integrated Equipment record and Maintenance Management System (TAERS) , out-of-service time is recorded only if the vehicle is delayed for more than 48 hours in being worked on, or if an essential repair part is not in stock when required. Vehicle jackets (the record of vehicle maintenance and repair cost summaries and preventative maintenance data) do not contain the time out-of-service, but rather the time the vehicle was actually

being worked upon. The amount of time that a vehicle is out-of-service, as the model envisages, must include all the out-of-service time. A full year's daily operations reports were analyzed for a unit based in Viet Nam. This examination of a composite group of vehicles indicated the out-of-service situation for the unit at eight o'clock each day. The population of vehicles varied from a high of 209 to a low of 186, with a mean of 197.8. The out-of-service vehicles varied from a high of 68 to a low of 16, with a mean of 32.75. The average daily unavailability was 16.58 percent. However, none of the time out-of-service connected with the fifth echelon rebuild was included in this daily average. The analysis did serve to give a fuller appreciation for the vagaries and random actions of the unknown production function. But the data did not lend themselves to time series study as there was no way to determine which of the approximately two hundred vehicles were the ones listed as out-of-service on the daily reports.

An approximation of the appropriate coefficient was generated by the following technique. Making the assumption that all the observed vehicles were of ages zero to ten years, and of equal increments, and that the population of vehicles in the unit that is being investigated is in the average age steady state, then the observed out-of-service rate will represent the sum of the interval from 4.5 years to 5.5 years of the average age maintenance function. It is necessary to account for the time required for fifth echelon rebuild. If we assume that there will be a four month out-of-service period each five years associated with the fifth echelon rebuild, then 6.57 percent must be added to the observed rate. The model shows the preventative maintenance time as a separate factor; therefore, this time must be

subtracted from this total. Using these assumptions a value for the coefficient can be derived as follows

.1658	Observed average daily out-of-service rate
.030829	Less preventative maintenance
.134971	
.0657	Plus fifth echelon out-of-service
.200671	

$$.200671 = \int_{4.5}^{5.5} \frac{1}{10} \int_{1}^{10} \frac{1}{10} \int_{0}^{10} e^{-a_m - b_m /t} \, dt\,dx\,dy$$

In the investigation, the maximum out-of-service rate observed on any one day was 36.5 percent. If we assume that the maximum attainable is 40.0 percent, then the value of a_m will equal .9183. With this value, the integral can be evaluated and the value of b_m established. In this case, $b_m = 2.161$.

The Repair Parts Function

The location of data for the derivation of the coefficients for the repair parts function presented the same difficulties as did the data for the maintenance function. A host of data is collected showing the amount of repair parts needed each year. However, identifying the costs associated with the age of a specific vehicle is an almost impossible task. In the absence of empirical data a similar technique as was used for the maintenance function can generate an approximation for the correct parameters.

From a report furnished by the Army Materiel Command Logistics Data Center for 200 5 ton M 52 trucks that were located in Viet Nam for at least nine months, it was possible to extract eighteen samples for vehicles that were two, three, and four years old. Unfortunately, there were no trucks listed under the age of two years or over five years.

A straight regression analysis of the data was not significant for the functional form postulated. Since both Smith and Drinkwater found that the postulated form was the correct form in empirical analysis of data that covered the entire age spectrum, the regression was discarded.[43,44] However, by taking the average cost of the vehicles over the three years represented, it is possible to derive coefficients that can be used. By setting the average cost of repair parts over the three years equal to the formula for the average age cost, a value for the coefficient is derived. First, it is necessary to assume an asymptotic value for the cost. In this case, the value represented by the repair-scrap decision is used. It is normal for the maintenance section of each service to set an arbitrary percentage which is used to determine if it is economical to repair a vehicle that has suffered some maintenance malfunction. Although this is normally a one time action, it is believed that it will serve the same purpose for this model. Therefore, assuming that the asymptotic value of the average cost per year will be a certain percentage of the acquisition value will establish the coefficient value of a_r. In this case we have chosen to use the repair-scrap value of .65. This can be interjected directly into the model or translated into exponential form. For simplicity, the computer model was developed with the constant term factored out of the exponential relationship; therefore, the desired coefficient would be the decision value. The eighteen vehicles of

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[43] Vernon L. Smith, "Economic Equipment Policies: An Evaluation," Management Science, (Vol. 4, 1958), p. 27.

[44] R. W. Drinkwater and N. A. J. Hastings, "An Economic Replacement Model," Operational Research Quarterly, (June 1967), p. 20.

each age averaged \$3,072 per year for the repair parts cost. The acquisition value for the vehicles is \$16,800 times the .65 gives \$10,920.

$$3{,}072 = \left(\frac{1}{3}\int_1^3 \frac{1}{4}\int_0^4 e^{-b_r/t}\,dt\,dx\right) 10{,}920$$

which solves to give a value of 1.624 for b_r.

Other Factors

In addition to the coefficients for the three subordinate functions of the model, the value of the personnel cost inflator, inflator for the repair parts cost, and acquisition value of the replacement items will be needed before a solution for the model is possible. For the personnel inflator, a factor of 2.704 is recommended. This was derived by using the total cost of direct labor at the rate of \$6.50 per hour. Of this rate per hour, 54 percent, or \$3.51 represents the cost per hour of the direct labor. Indirect supervisory and support personnel represents 21.7 percent or \$1.41. Repairs and utilities represent 7.7 percent or \$ .50, and amortization of tools and ancillary equipment represent 16.5 percent or \$1.07.[45]

This factor assumes that the average man is available 74 percent of the normal work period, the remaining being devoted to leave, liberty, sickness, and other military duties. For the example, the annual pay rate of \$5,000 per man is used. These aggregations of diverse personnel ranks and rates, as well as other costs into a fixed

[45] Virginia W. Perry and others, "A Replacement Requirement Methodology for Procurement of Army Equipment," (Vol. II), U. S. Army Logistics Management Center, Fort Lee, Va., July 1967.

cost per hour may seem to be generating an excessive cost per hour for the cost of the operation and maintenance personnel. However, the detailed cost analysis performed by the Army Logistics Management Study revealed these percentages in their empirical work. Their work has been translated into the factor provided in order that the model can be used as the military pay scale changes.

When repair parts are purchased and placed in the supply system, there are costs incurred which should be charged to the maintenance of the supported equipment. The inflator for the stock, store, and issue cost of the repair parts will vary with the item cost and density of the supply requirement. The use of an inflator insures that the full cost of all submerged operationsare considered in the model. Although the Army's derivation excluded the high-dollar components from their inflator, a gross inflator has been included in the model to compensate for the decreases occasioned by the exclusion of the rebuild costs from the continuous repair parts function. For the example a factor of 1.50 is used.[46]

An acquisition cost of $16,800 is used for the M 52 truck used in the example.[47] This cost should include all costs of contract preparation and implementation.

Summary

[46]This factor was derived in the U. S. Army Logistics Management Center Study for the ½ ton truck from a study titled "Cost of Supply Operation," by Harbride House, Inc., Report #2, undated, Vol. II, p. 9.

[47] Cost figures used were given by the Commodity Manager, Headquarters U. S. Marine Corps at a personal interview dated 22 March 1968.

Data were not obtained which would allow the validation of the functional relationships by empirical analysis. However, the forms of the relationships have been established in other works. By using known data for a fixed period, it is possible to approximate the coefficients in order that the model may be exercised. Other factors used in the model were taken, for the most part, from an existing study. With the coefficients and factors provided in this manner, the model can be exercised.

CHAPTER V

EXERCISING THE MODEL

"In judging the suitability of a certain model to a particular case, the important matter is to judge whether the decisions based on the use of the model is likely to be sensitive to the various ways in which the assumptions depart from the facts of the case."[48]

General

In order to solve the models developed in the foregoing chapters, two methods were considered -- the use of calculus and numerical iteration. The use of calculus produced such a complex equation for the first derivative that a computer program would be needed to obtain a solution. An iterative numerical evaluation, again with a computer solution, also proved to be quite complex. By combining the two methods using the basic definition of the derivative and a numerical iteration root finder, a computer program was developed which would solve the model with a minimum of computer time. The computer program was also designed to allow the parameters to be varied for the sensitivity analysis. (See Appendix I)

Factors and Coefficients

For use with the M 52 truck example, the following parameters were used for the analysis:

[48] Eugene L. Grant and W. Grant Ireson, Principles of Engineering Economy, (New York: Ronald Press, 1960), p. 534.

SSI (Repair Parts) Inflator (R) = 1.5

Repair Parts Cost Function ($e^{a_r - b_r/t_o}$)

$a_r = c_3$ (Scrap/repair decision)

$= 16{,}800 \times .65$

$b_r = 1.641$

Personnel Inflator and Yearly Cost Per Man (P)

P = 2.704 times $5,000 = 13,520

Personnel Function ($a_1/x + b_1$)

$a_1 = 3.105$

$b_1 = 2.630$

$x = 30$ (an average unit size of 30 has been assumed for the example)

Maintenance Function ($d + e^{-a_m - b_m/t_o}$)

$d = .030829$

$a_m = .9163$

$b_m = 2.161$

Discount Rate (k)

$k = .10$

Acquisition Cost (c_3)

$c_3 =$ $16,800

Batch Attrition (y_3)

$y_3 = .06$ (1.2 times .05)

Continuous Attrition (y_3)

$y_3 = .05$

Salvage Value ($y_4 - y_5 t_o$)

$y_4 = .615$

$y_5 = .071$

These parameters were introduced into the model program, Appendix I, and the model exercised under both the batch procurement and continuous procurement mode.

Solution Values

In the batch procurement mode, the model was exercised and a solution was not found. The function values were printed and plotted

and the plot showed that there was a continually decreasing function for the model with the ten percent discount rate. The discount rate was varied from zero to fifteen percent. At zero rate of discount, the solution value was .952 years. A solution value of 1.501 years was found at a discount rate of nine percent. These results would indicate that this vehicle should be replaced every year if discounting is not used, every year and one-half with a discount rate of nine percent, and that the vehicles should be maintained until they are replaced for technological reasons or casualty attrition if a discount rate of over nine percent is used.

In the continuous mode, the same situation arose. There was no solution at ten percent discount rate. A zero discount rate solution of 1.408 years was found and 2.185 years at four percent. There was no solution value found for a discount rate over four percent.

With zero salvage value, the batch mode found a solution at 1.487 with no discount and truncated at 5 percent with a value of 2.200. The continuous mode found a solution at 1.489 with no discount and truncated at 5 percent with a value of 2.204.

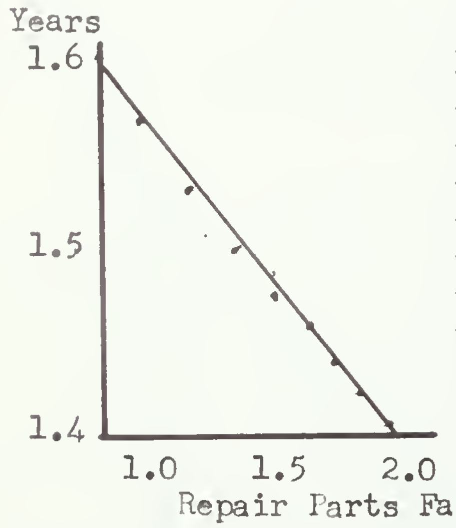
Sensitivity Analysis

To test the sensitivity of the model, the introduced parameters were varied within a reasonable range. The base case is the batch procurement with zero discount rate. The results of these tests are as follows:

Stock, Store, and Issue Inflator (R)

The inflator was varied from 1 (no inflation) to 2.0 (100% inflation) and the resulting solutions varied from 1.606 to 1.396

years. These results indicate that a 100% increase in the cost of repair parts causes a 15% decrease in the optimum replacement cycle. For this range, the relationship is almost linear as shown in this graphical presentation:



Years	R
1.606	1.000
1.579	1.100
1.554	1.200
1.530	1.300
1.508	1.400
1.487	1.500
1.467	1.600
1.448	1.700
1.430	1.800
1.412	1.900
1.396	2.000

Table 1

The results show that this factor is important in the determination of the economic service life.

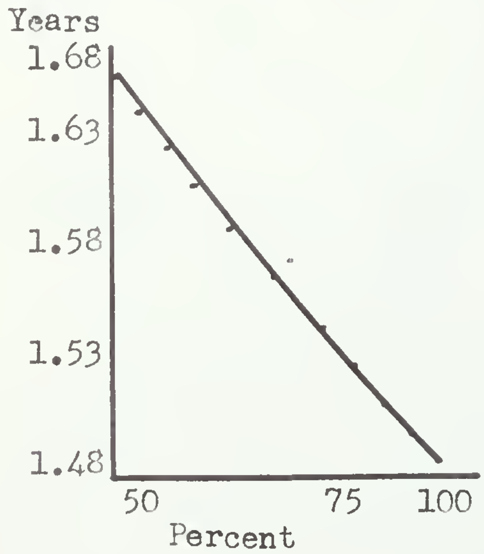
Repair Parts Cost Asymptote (e^{a_r})

The asymptote for the repair parts cost function was addressed as a variable of the maximum cost per year and was equated to the scrap-repair decision. Values for the policy variable for the scrap-repair decision of 45 percent to 100 percent of the acquisition value were used. The solution values decrease as the maximum cost per year is allowed to increase. However, when it is appreciated that lowering the scrap-repair decision will increase the attrition rate, this analysis loses much of its possible significance. Table 2 shows a graph and the solution values.

Repair Parts Coefficient (e^{-b_r/t_o})

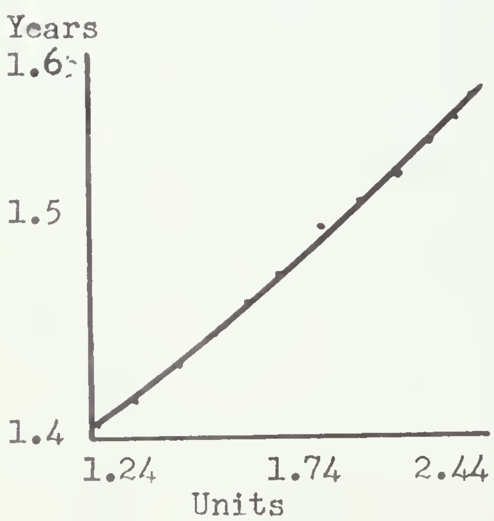
The repair parts coefficient was varied from 1.241 to 2.44.

The parameters were computed on costs which represented an increase of 33 percent in the cost of the parts at year five. The 33 percent increase in costs produced a 20.5 percent decrease in the replacement cycle time. The repair parts cost coefficient is significant in the determination of the economic service life of these vehicles. A graphical presentation of the solution values is shown in Table 3.



Years	a_r e
1.703	.450
1.677	.500
1.652	.550
1.629	.600
1.606	.650
1.585	.700
1.565	.790
1.546	.800
1.528	.850
1.511	.900
1.495	.950
1.479	1.000

Table 2



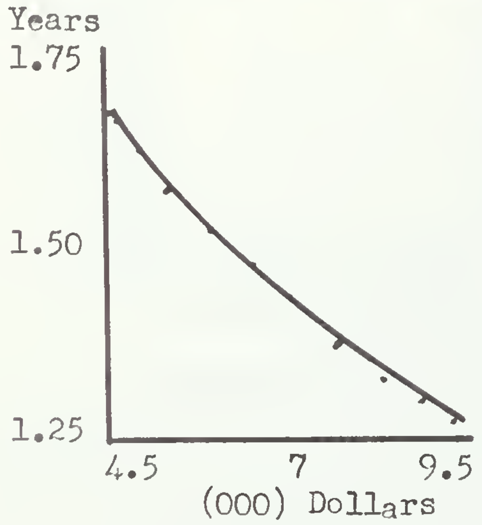
Years	b_r/t_o
1.418	1.241
1.436	1.341
1.453	1.441
1.470	1.541
1.487	1.641
1.503	1.741
1.519	1.841
1.535	1.941
1.550	2.041
1.579	2.241
1.608	2.441

Table 3

Personnel Costs (P)

The average yearly wage for personnel was varied from $4,500 to $9,500. The solution values varied from 1.689 to 1.251 years. Doubling the wage rate produced a 24.3 percent decrease in the

solution value. This result would indicate that the wage rate is a minor, but significant determinant in the economic service life of these vehicles. Table 4 shows the solution values for this parameter.

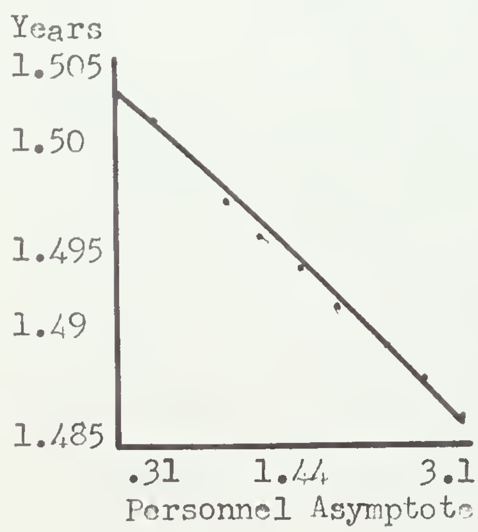


Years	P
1.689	4500
1.606	5000
1.559	5500
1.505	6000
1.457	6500
1.414	7000
1.375	7500
1.340	8000
1.308	8500
1.277	9000
1.251	9500

Table 4

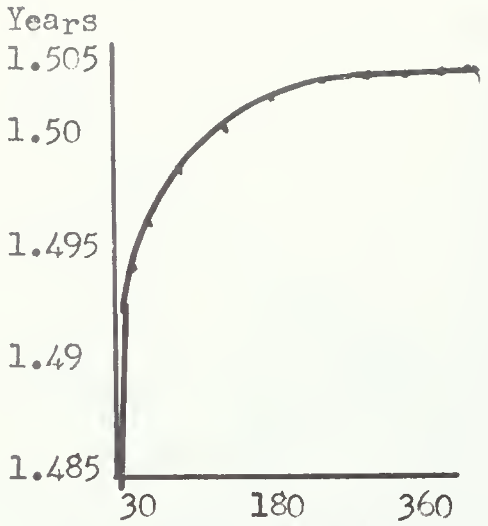
Personnel Function Intercept (a_1/x)

The personnel function intercept was varied from .31 to 3.425 and there was little change in the solution value. This is due to the fact that the intercept term is divided by the number of vehicles in the normal sized unit for this type vehicle. The number of vehicles in the unit was varied from the example value of 30 to 360 with similar results. The solution values for the intercept term are shown in Table 5 and those for the number of vehicles in Table 6.



Years	$a_1/30$
1.503	.310
1.502	.621
1.500	.931
1.498	1.245
1.496	1.555
1.494	1.865
1.492	2.175
1.490	2.485
1.488	2.795
1.487	3.105
1.485	3.425

Table 5

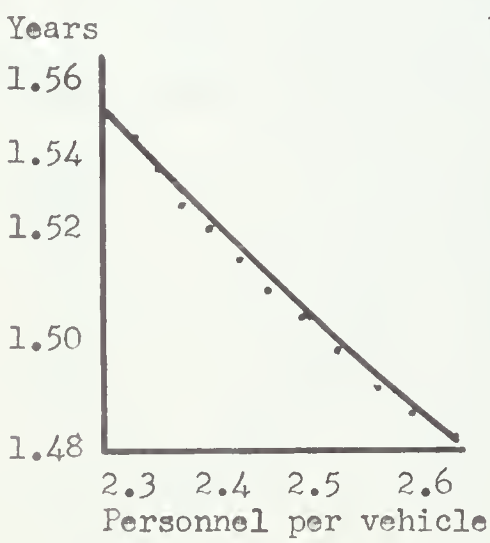


Years	3.105/x
1.487	30
1.496	60
1.499	90
1.501	120
1.502	150
1.502	180
1.503	210
1.503	240
1.503	270
1.503	300
1.503	330
1.503	360

Table 6

Personnel Function Coefficient (b_1)

The personnel function coefficient was varied from 2.31 to 2.631 and it produced only a minor change in the replacement cycle. This result was unexpected in view of the proportion of the costs which are represented by personnel costs. The solution values for these parameter values are shown in Table 7.



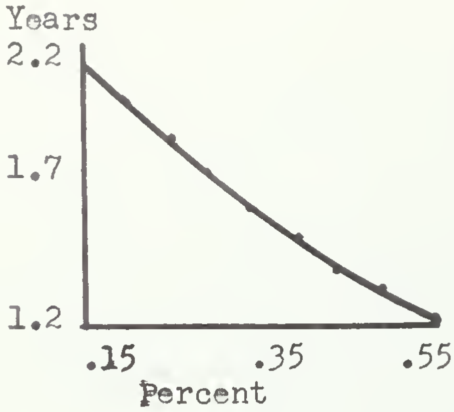
Years	b_1
1.547	2.310
1.541	2.342
1.534	2.375
1.528	2.407
1.522	2.439
1.516	2.472
1.510	2.503
1.504	2.535
1.498	2.567
1.492	2.599
1.487	2.631

Table 7

Maintenance Function Asymptote (e^{a_m})

The model was formulated on the basis of a maintenance function which would have a maximum out-of-service rate. The maximum was varied from 15 percent to 55 percent with the result that the solution value decreased from 2.137 to 1.268 years. The solution values are

shown below in Table 8.

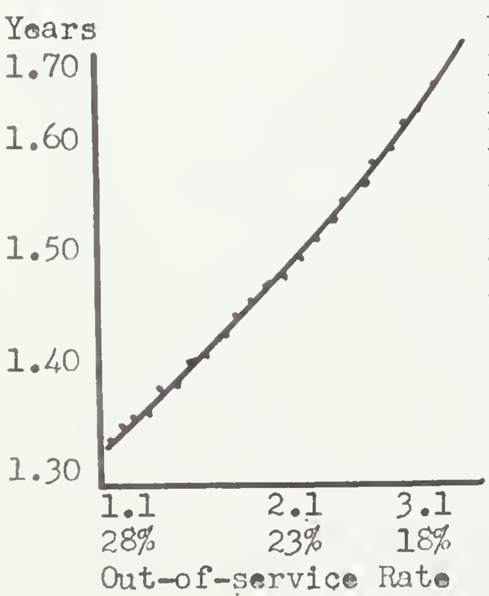


Years	e^{a_m}
2.137	.15
1.875	.25
1.764	.30
1.600	.35
1.487	.40
1.401	.45
1.329	.50
1.268	.55

Table 8

Maintenance Function Coefficient ($-b_m/t_o$)

The maintenance function coefficient was varied from 1.161 (28 percent average out-of-service rate) to 3.161 (18 percent average out-of-service rate) with the results that the solution values increased from 1.339 to 1.687 years, a 20 percent increase. This parameter, as would be expected, is a significant determinant in the economic service life of these vehicles. This function is a crucial factor in determining the total inventory required to meet the constant operating requirement. Table 9 shows the solution values for this parameter.

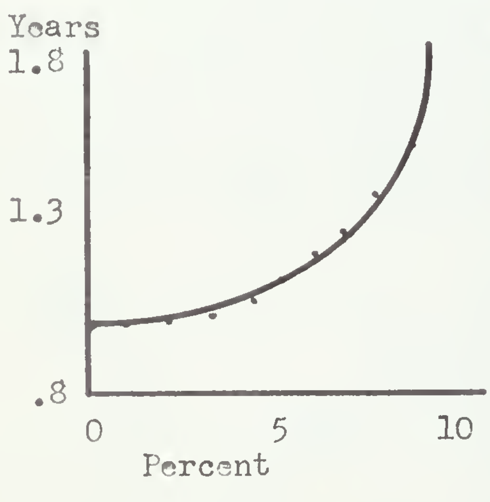


Years	b_m/t_o
1.339	1.161
1.345	1.261
1.354	1.361
1.367	1.461
1.381	1.561
1.397	1.661
1.413	1.761
1.431	1.861
1.449	1.961
1.468	2.061
1.487	2.161
1.506	2.261
1.525	2.361
1.544	2.461
1.563	2.561
1.602	2.661
1.620	2.761
1.639	2.861
1.659	2.961
1.678	3.061

Table 9

Discount Rate (k)

The discount rate was varied from zero to fifteen percent in both the batch and continuous mode. In the batch mode, solutions were found for values through 9 percent, but none were found for 10 percent through 15 percent. In the continuous mode, solution values were found for discount values through 4 percent, but none for 5 percent through 15 percent. On the surface, this would appear to be a curious anomaly of the model. However, investigation reveals that the discount rate controls the entire equation as soon as the maintenance function and repair parts cost functions start to assume a uniform shape after passing the inflection point. A plot of the function value for the truncated range shows an ever decreasing value, with no maximum or minimum over the range. Table 10 shows the solution values for the batch mode. The relationship is asymptotic to a value of ten percent.

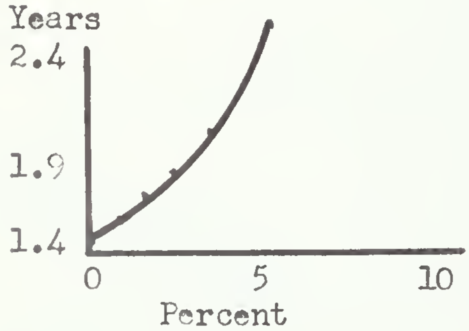


Years	K
.952	.00
.982	.01
1.012	.02
1.049	.03
1.089	.04
1.135	.05
1.191	.06
1.260	.07
1.353	.08
1.501	.09

Table 10

The same relationship holds for the continuous mode; however, the asymptotic value is five percent. Table 11 shows the solution values for the continuous mode. These results show that the current military policy of maintaining these vehicles until they are replaced for reasons of attrition or technological advance is a correct policy. It also shows that the use of discounting techniques serves to make an

existing system more competitive when comparing them with newer systems. Acceptance of a nine percent discount rate, rather than the normal Department of Defense ten percent, would indicate that these M 52 vehicles should be replaced every year and one-half instead of the current eight to ten years.

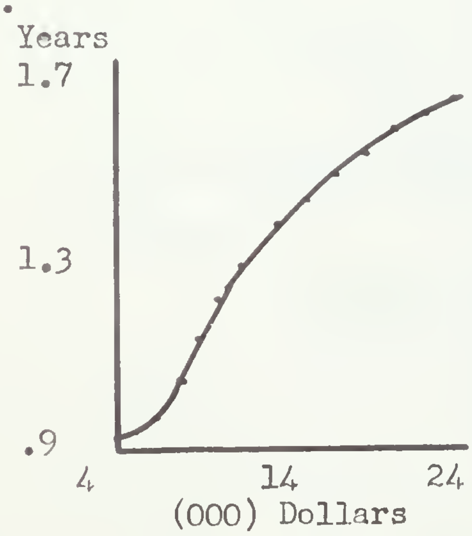


Years	K
1.408	.00
1.513	.01
1.650	.02
1.844	.03
2.185	.04

Table 11

Acquisition Cost (c_3)

The acquisition cost was varied from $4,000 to $24,000 and the solution values ranged from .908 to 1.661 years. The cheaper vehicles should be replaced more often than the expensive vehicles. This result is what one would normally expect. Since the acquisition cost was included as a factor in the repair parts function, these results should be quite accurate. Table 12 shows the solution values for this parameter.



Years	c_3
1.661	24,000
1.618	22,000
1.572	20,000
1.520	18,000
1.463	16,000
1.399	14,000
1.326	12,000
1.246	10,000
1.152	8,000
1.041	6,000
.908	4,000

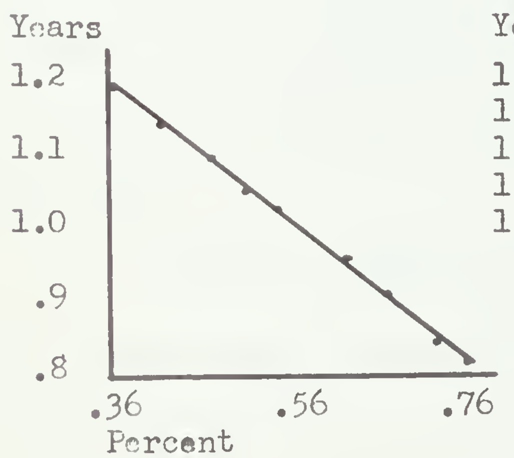
Table 12

Salvage Value Function ($y_4 - y_5\ t_o$)

The salvage value parameters were varied in a reasonable range. The uniform rate of decrease per year (y_5) was varied from .051 through .096 per year. The resulting solutions show that this parameter is not significant in the determination of the model. Table 13 shows these solution values. The initial reduction in value (y_4) was varied from .36 to .76. Here there was a significant effect on the solutions. The solution values are shown in Table 14.

Years	y_5
.960	.051
.960	.056
.959	.061
.959	.066
.959	.071
.958	.076
.957	.081
.956	.086
.956	.091
.955	.096

Table 13



Years	y_4
1.180	.36
1.137	.41
1.094	.46
1.050	.51
1.005	.55
.957	.61
.909	.66
.858	.71
.803	.76

Table 14

Further Considerations

In the development of the model, it was noted that it was not possible to identify the personnel function in relation to the age of the supported vehicles. This feature apparently has not received consideration because of the institutional constraints of the military personnel system. However, in view of the drastic changes which are

indicated by the model's solution if there were to be very minor modifications of the discount rate, it is believed that some investigation of the possible effects of a variable personnel rate as a function of the age of the vehicle should be made. Although the criteria manual is not specified in terms of an average age of the vehicles, it is obvious that some average age must have been imputed in the development of these allowances. For it is clear that it will take more personnel to maintain and repair a ten year old vehicle than one only one year old. A vehicle of two equivalents such as the M 52 of our example requires two operators to be assigned for each item. Therefore, only the portion of the coefficient remaining when these two are subtracted could be subject to a variable rate with age. If we assume that about eighty percent of the amount remaining is subject to a variable with age in the same relationship as the repair costs function, we could formulate an expression which would indicate the variable. The use of the following relationship will allow a reasonable change in the amount of personnel required to support the vehicles as a function of the age of the vehicle

$$b_1 = 2.119 + 1.08336 \left(\frac{1}{t_o} \int_0^{t_o} e^{-b_r/t_o} dt\right)$$

The computer program was modified to accommodate this change and the example tested for both the batch and continuous modes. The batch case gave solution values of .892 for the zero discount rate, 1.175 for the ten percent discount rate, and truncated at fourteen percent with a value of 1.557 years. The continuous mode gave solution values of 1.144 for the zero discount rate, and truncated at eight percent with a value of 1.994 years. This modification would then change the situation for the batch procurement to one where the economical

replacement cycle would be about one year instead of maintaining the vehicles until replaced for some other reason. In the continuous mode, there was a significant increase in the discount rate where the model truncated. This increase to eight percent almost reaches the normal Department of Defense rate.

Summary

The model was exercised with the parameters which were derived for the example. Solutions were obtained by the use of a computer program, Appendix I, which uses the basic definition of the derivative and numerical iterative methods to find a solution for the equation. Under both the batch and continuous modes, the model indicates that the present military vehicle replacement policy is the correct economic solution. The modification to the personnel function, allowing the number of personnel to vary with the age of the equipment, shows that there is a strong possibility that there should be a drastic revision in the replacement policies. The batch procurement model was tested for sensitivity in all parameters. Each parameter showed that there was some effect, but the number of vehicles in the units and the personnel intercept were almost insensitive. The annual rate of decrease in the salvage function also plays little role in the model. As would be expected the major influence was shown by the maintenance function.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

"The consequences of an inadequate replacement policy for the firm are potentially disastrous."[49]

General

William T. Morris states that an inadequate replacement policy for the firm can have dire consequences. For the firm, it would be the loss of profit or good will which would be the consequence. In the military situation, the consequence would be the loss of effectiveness. If the loss of effectiveness happens at some crucial point in combat operations, the dire consequence can easily be catastrophic. It is impossible to quantify the costs which may be associated with such an occasion. Therefore, in this treatment an attempt is made to formulate a replacement model which minimizes the chance of the loss of effectiveness below a minimum standard. In order to do this, it is necessary to modify certain usual military usages. Normally, when we speak of a military requirement, it is the Table of Equipment or Table of Allowances to which we refer. If these allowances are thought to be the requirement, little can be done to preclude some derogation of the effectiveness. When we use the mission as the basis for the requirement, it is possible to define the problem in terms which allow us to seek a solution which can overcome the possible loss of effectiveness.

[49] William T. Morris, The Analysis of Management Decisions, (Homewood, Ill: Richard D. Irvin, 1964), p. 193.

The model is developed to provide a minimum acceptable operating requirement in terms of the capability to perform the mission. The model computes the cost of the entire system and provides the minimum average cost in terms of the replacement cycle, subject to maintaining the minimum acceptable operating level.

Batch Procurement

In many cases of low total inventory, the economics of bulk purchasing will dictate that the total requirements be purchased initially. When this situation is encountered, a more rapid replacement cycle will be required to ensure that the minimum acceptable operating level is maintained. From the example, it is apparent that the replacement cycle may be reduced as much as one-third from that of a continuous procurement. However, it is important to realize that this model is designed for tactical equipment. If there is some alternative use for these items, which will accept some lower minimum operation level, then a two-step process may well be possible. Using the alternative use value as the salvage value for the batch situation, a solution for the batch mode could be found. The items could then be transferred to the alternative use and the continuous mode or batch with a lower operation level could be made.

It is apparent from the model that there may well be such a limited service life for tactical military equipment that the question of rebuild will not arise. The question of rebuild for these items would only be considered for those cases where there is some alternative use.

Continuous Procurement

The continuous procurement mode of the model provides a solution for the situations where the replacements are programmed over the service life of the equipment. This model averages the age of the equipment and thereby allows for the higher availability and lower operating cost associated with the age of the equipment. With the parameters derived for the example, the model shows that under the continuous procurement mode the equipment should be operated and maintained until they are replaced by mortality or obsolescence. However, since these parameters are approximations, it would appear to be prudent to conduct detailed investigations to ascertain more accurate parameters. Detailed day to day out-of-service and cost data is needed to derive the maintenance and repair cost functions in their exact forms. It is possible that a refinement of these two functions will produce a solution for both modes at the Department of Defense discount rate of ten percent.

Personnel Requirements

The dramatic change produced by the modification to the personnel function to attempt to account for the age of the equipment indicates that a serious investigation of this feature should be conducted. The approximation used in the analysis produced a solution value for the batch mode and truncated at an eight percent rate for the continuous mode. I believe that this result clearly indicates that the personnel criteria should be predicated on the age of the assigned equipment. Although I have confidence in the approximations, an exact definition of the relationship would serve to provide a more accurate model. Such

a change would not pose significant changes to the existing organizational framework, for the criteria based on the average age of the supported equipment would be standard once the replacement cycle was established.

Summary

The model developed in this paper offers a means to preperly compute the replacement cycle for equipment that has a minimum acceptable operating level. It considers the system in its entirety and optimizes both the total quantity of equipment required as well as the replacement cycle time. It clearly points out two areas that require additional investigation. The identification of the out-of-service time and repair parts cost on a daily basis will be needed to derive properly the parameters of the functions. In the interim, approximations can be developed through the use of the techniques used in this paper. Identification of the personnel requirements as a function of the age of the supported equipment is also required. Again, the approximation technique could be used until more accurate information is available.

Although the model, with the derived parameters, shows that the current military replacement policies are those of the economic service life, small changes in the parameters could produce drastic changes. For this reason, I strongly recommend further investigation to determine the accurate parameters. In the military situation, the consequences of an inadequate replacement policy is potentially catastrophic. The use of the model presented in this paper will tend to overcome this shortcoming and provide replacement decisions which will insure that the risk is minimized. We normally consider tactical military vehicles a mundane

problem when compared to the more sophisticated weapons systems. But, the consequences of an inadequate vehicle replacement policy can be disastrous! Every effort should be expended to produce the needed information in order that a crucial test of our current replacement policies can be made.

Appendix I

COMPUTER PROGRAM FOR THE MODEL

The following computer program was developed in Fortran IV to provide a solution to the cost equation in either the batch or continuous mode. Some of the program was designed to facilitate the sensitivity analysis, but it will not inhibit the use of the program for simple solutions.

```
      PROGRAM ROOTY
CDUPBEG
      Common Y5,Y6,Y7
      Common BXX
      Common Y1,Y2,Y3,Y4
      Common XP7
      Common DATA (30)
      Common NDAT, INDEX
      Common AM
      Common RLOW, RUP, RERR
      Common R, AR, TO, BR, P, A1, B1
      Common BM, XK, C3, PA1, PB1
CDUPEND
      DCF
    3 CALL RDCARDS
      XO = RLOW
      XF = RUP
      ERR = RERR
   30 CONTINUE
      X = ROOTF(XO, XF, ERR, DCF)
      IF(X, GT, XF) 1, 2
    1 CONTINUE
      PRINT 10
   10 FORMAT(*NO ROOT FOUND)
      GO TO 3
    2 CONTINUE
      AA = CF(X)
      BB = CF(X + .01)
      IF(BB, GT, AA) 5, 4
    4 XF = X - .001
      GO TO 30
    5 CONTINUE
      PRINT 20, X, R, AR, BR, P, A1, B1, AM, BM, XK, C3
   20 FORMAT(1X, F10.3, 4X, 10F10.3)
      GO TO 3
      END
```

```
      FUNCTION DCF(X)
      DATA(XINC=.001)
      DCF=(CF(X+XINC)-CF(X))/XINC
      END
      SUBROUTINE RDCARDS
CDUPDIM
        NO LISTS
      DATA(ISW=1)
      GO TO (1,2,50), ISW
    1 CONTINUE
      XP7= 1.08336
      ISW= 2
      READ 10,RLOW,RUP,RERR,C3,AM
      READ 10,Y1,Y2,Y3,Y4,Y5
      PRINT 999, Y1,Y2,Y3,Y4,Y5
  999 FORMAT(10F10.3)
   10 FORMAT(8F10.0)
      READ 10, R,AR,BR,P,A1,B1,BM,XK
      PA1=P*A1
      PB1=P*B1
      RETURN
    2 CONTINUE
      GO TO 10
      READ 20. NDAT,(DATA(I),I=1,NDAT)
   20 FORMAT(I5,15F5.0)
      READ 20,INDEX
      ISW=3
   50 CONTINUE
      ICTR=ICTR+1
      IF(ICTR-NDAT)21,21,45
   45 ICTR=0
      GO TO 2
   21 X=DATA(ICTR)
      GO TO (22,23,24,25,26,27,28,29,31,32),INDEX
   22 R=X
      GO TO 30
   23 AR=X
      GO TO 30
   24 BR=X
      GO TO 30
   25 P=X
      GO TO 30
   26 A1=X
      GO TO 30
   27 B1-X
      GO TO 30
   28 BM=X
      GO TO 30
   29 XK=X
      GO TO 30
   31 C3=X
      GO TO 30
   32 AM=X
      GO TO 30
   30 CONTINUE
```

```
      RETURN
   40 CONTINUE
      STOP 1604
      END
      FUNCTION ZILCH(T)
CDUPDIM
        NO LISTS
      ZILCH=EXPF(-BXX/T)
      END
      FUNCTION BILCH(EP,V)
CDUPDIM
        NO LISTS
      EXTERNAL ZILCH
      DIMENSION ARR(400)
      A=ZINT(ZILCH,EP,V,VR,ARR,100)
      BILCH=A
      END
      FUNCTION CILCH(T)
CDUPDIM
        NO LISTS
      Q1=BR/T
      Q2=Q1+2
      Q3=Q2*Q2
      Q4=Q3*Q3
      W1=EXPF(-Q1)/Q2
      W2=1.+2./Q3+(4.-4.*Q1)/Q4+(12.*Q1*Q1-32.*Q1+8)/(Q4*Q3)
      CILCH=W1*W2
      END
      FUNCTION CF(X)
CDUPDIM
        NO LISTS
      DIMENSION BARR(200)
      EXTERNAL BILCH
      EXTERNAL CILCH
      TO=X
      TO=TO
      BXX=BM
      XM=.030829
      SP=1.0/(1.0-XM-Y1*EXPF(-AM-BM/TO)-
                       Y2*EXPF(-AM)*BILCH(1.0,TO)/TO)
      VX=SP*EXPF(-XK*TO)
      VY=PA1/30.0+PB1
      Q1=ZINT(CILCH,1.0,TO,VR,BARR,100)
      VZ=Y1*R*AR*C3*CILCH(TO)+Y2*R*AR*C3/TO*Q1
      VW=(C3/TO)*(1.0+Y3*TO-Y4+Y5*TO)*VX
      CF=VX*(VY+VZ)+VW
      END
      SUBROUTINE COMPU
CDUPDIM
        NO LISTS
      DIMENSION WORKS(400)
      DATA(XDEL=.1)
      FUNCEX
      NCELLS=400
      X=RLOW
```

```
3 CONTINUE
  Z=CF(X)
1 FORMAT(2F10.2,4X,3F15.10,4X,F10.3)
  PRINT 1. X,Z
  IF(X-RUP)2,2,4
2 X=X+XDEL
  GO TO 3
4 CONTINUE
  END
  END
```

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